ASYNCHRONOUS COUPLING OF ICE-SHEET AND ATMOSPHERIC FORCING MODELS

by

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ABSTRACT

Asynchronous coupling schemes between ice sheet and atmospheric forcing models are evaluated for use in long-term ice-age simulations. In these schemes the ice sheet and atmospheric forcing are run together for short synchronous periods \( (T_s) \), alternating with longer asynchronous periods \( (T_A) \) during which the ice sheet is run with atmospheric information extrapolated from the previous synchronous period(s). Two simple ice-sheet models are used that predict ice thickness as a function of latitude, and the atmosphere is represented by a prescribed pattern of net annual accumulation minus ablation. The pattern is shifted vertically to represent long-term orbital variations, stochastic inter-annual weather variability and ice-sheet albedo feedback.

Several asynchronous schemes are evaluated by comparing results with those from fully synchronous runs. The best overall results are obtained using a scheme in which the forcing during each asynchronous period is linearly extrapolated from its means in the previous two synchronous periods. Differences from the synchronous results are caused primarily by poor sampling of the stochastic forcing component, which exaggerates the stochastic ice-sheet fluctuations. We examine how these errors depend on \( T_s \) and \( T_A \) and outline implications for GCM ice-age simulations.

MODEL DESCRIPTION

Ice-sheet and atmospheric forcing

The models used here have found standard usage in many ice-age studies; for brevity only a general description is given below, with the model components shown schematically in Figure 1. For most of our results we used the perfectly plastic ice-sheet model (e.g. Weertman, 1976), in which the ice-sheet profile versus latitude is constrained to be parabolic. The bedrock depression below the ice is assumed to be isostatic, and the northern ice-sheet tip is fixed at the Arctic Ocean shoreline. The ice sheet varies in size according to the surface accumulation minus ablation on its southern half, always keeping the parabolic profile. As in Weertman (1976), the atmospheric forcing is simply a

![Fig. 1. Schematic cross section of the ice sheet. \( h_0 \) is the elevation of the snowline at the Arctic Ocean shoreline.](image-url)
prescribed pattern of net annual accumulation minus ablation, which is shifted vertically to represent long-term orbital perturbations, stochastic inter-annual weather variability and ice-sheet albedo feedback. The forcing is shown in the figures below in terms of $h_0$, the elevation of the snowline at the Arctic Ocean shoreline, given by

$$h_0(t) = h_{\text{orb}} + h_{\text{alb}} + h_{\text{sto}}.$$  

$h_{\text{orb}}$ is a long-term sinusoidal forcing analogous to the effects of orbital perturbations on the atmospheric climate, and its form for each run can be seen in the figures below. The second term $h_{\text{alb}}$ represents the albedo feedback of the ice sheet, and lowers the snowline in proportion to the current ice-sheet extent.

The third term $h_{\text{sto}}$ in Equation (1) represents the year-to-year variability due to inter-annual weather variations, and is important for the present study since it introduces the possibility of sampling errors in asynchronous coupling schemes (Harvey, 1986; Hasselmann, 1988). In reality and in GCMs, the inter-annual variability of storm tracks, temperature and other atmospheric fields causes the net mass balance on ice surfaces to fluctuate from year to year, with amplitudes on the order of one-tenth of the glacial-interglacial changes. If in an asynchronous run the synchronous period $T_s$ is set too short, one or two abnormal weather years can skew the extrapolated forcing used throughout the next asynchronous period. $h_{\text{sto}}$ is assigned a random value annually during synchronous periods, distributed normally with zero mean and standard deviation of 100 m and with no correlation from year to year. The amplitude of 100 m is derived from observed summer temperature variability in Oort and Rasmussen (1971) and winter snow-cover variability in Wiesnet and Matson (1979), but the most appropriate value is uncertain as discussed below (cf. Oerlemans, 1979).

If the southern half of the ice sheet lies either entirely below or entirely above the snowline, the assumption of a fixed parabolic profile is no longer valid (Weertman, 1976; Birchfield, 1977). We encountered this situation in only one type of run, in which the ice sheet vanishes periodically; as a nascent ice sheet starts to grow it remains entirely above the snowline for a few thousand years, and vice versa as a shrinking ice sheet vanishes. This caused spurious effects in our results, so for that type of run we used a more general diffusive ice-sheet model. (For all other types of runs the results obtained from the two models agreed well.) The diffusive ice-sheet model has been used with variations in many ice-age studies (e.g. Birchfield and Grumbine, 1985; Hyde and Peltier, 1987). It is based on a vertically integrated ice-flow law, and predicts ice thickness as a general function of latitude. We take the bedrock depression to be isostatic, constrain the northern ice-sheet tip at the Arctic Ocean shoreline, and use the same atmospheric forcing pattern as Birchfield and Grumbine (1985).

**Types of asynchronous coupling**

Our base-line coupling method is fully synchronous, with the "atmospheric state" $h_0$ computed from Equation (1) at every ice-sheet time step. This method is referred to as "synchronous" below, and differences from the synchronous results caused by other coupling methods are termed "errors".

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Fig. 2. (a) Solid curve: ice-sheet extent (distance from northern to southern tip) vs time using the diffusive ice-sheet model with synchronous coupling. Dotted curve: atmospheric forcing $h_0$ as defined in Equation (1). (b–d) Differences from Figure 2a using: (b) non-extrapolated asynchronous coupling; (c) extrapolated asynchronous coupling; (d) least-squares asynchronous coupling.

Fig. 3. As for Figure 2, except using the plastic ice-sheet model and smaller amplitude in the long-term sinusoidal forcing $h_{\text{orb}}$. 

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In our simplest asynchronous coupling scheme, the atmospheric state $h_g$ throughout each asynchronous period $T_{A}$ is taken as its mean value during the previous synchronous period $T_{S}$. This is referred to below as "non-extrapolated".

In a slightly more refined scheme, $h_g$ during each asynchronous period $T_{A}$ is obtained by linearly extrapolating in time from its mean values in the previous two synchronous periods. Harvey (1986) used a similar scheme for atmosphere-ocean coupling (his $T_{S}$), except that he extrapolated quadratically using the previous three synchronous values. Our linear scheme is referred to below as "extrapolated".

In an effort to minimize the sampling errors due to the $h_{fo}$ term in Equation (1), we experimented with a more complex form of the diffusive ice-sheet model as explained above. All the error curves in Figure 2 show large spikes when the ice sheet is decaying rapidly (ice extents < -500 km), because distortions in the forcing applied for one 990 year asynchronous period can have a relatively large effect when the ice is thin. When the ice sheet is large its relaxation time is on the order of 7000 years (Birchfield, 1977), and the uncorrelated distortions of several consecutive 990 year asynchronous periods usually cancel out before the ice sheet reacts much.

However, spikes of about half the size of those in Figure 2 occur using synchronous coupling alone, just by choosing different realizations of the random number sequence used for $h_{fo}$ (When the ice sheet is large, the differences between synchronous realizations are negligible.) Hence only about half of the spike amplitudes in Figure 2 is large enough to make the ice sheet vanish periodically, which necessitated the use of the random component $h_{fo}$ during many (about 40) synchronous periods. The mean values of $h_{fo}$ are then extrapolated to obtain $h_g$ through the next asynchronous period $T_{A}$. This method is referred to below as "least-squares".

RESULTS

Examples

Figures 2 and 3 show typical results for sinusoidal forcing $h_{orb}$, with $T_{S} = 10$ year, $T_{A} = 990$ year. The amplitude of $h_{orb}$ in Figure 2 is large enough to make the ice sheet vanish periodically, which necessitated the use of the random component $h_{fo}$ during many (about 40) synchronous periods. The mean values of $h_{fo}$ are then extrapolated to obtain $h_g$ through the next asynchronous period $T_{A}$. This method is referred to below as "least-squares".

In Figures 2b and 3b are systematically positive when the ice sheet is shrinking rapidly, because distortions in the forcing applied for one 990 year asynchronous period can have a relatively large effect when the ice is thin. The errors in the extrapolated method depend on the choices of $T_{S}$ and $T_{A}$. These errors are basically caused by inadequate sampling of the inter-annual variability $h_{fo}$ in Equation (1) during each synchronous period. In synchronous runs this term is felt by the ice sheet as white noise, and causes red-noise fluctuations in our model ice sheet of about 3 km (cf. Oerlemans, 1979; Oerlemans and Van der Veen, 1984; it also causes a reduction in mean size of the ice sheet). With asynchronous coupling, the forcing during $T_{A}$ is based on averages taken over the previous two synchronous periods $T_{S}$. These averages depart randomly from the true mean climate by an amount $(1/T_{S})^1/2 \times$ the inter-annual variability, since the standard deviation of the mean of $N$ random samples is $(1/N)^1/2 \times \sigma_{h_g}/\sigma_{h}$ (where $\sigma_{h_g}$ is the inter-annual variability in the forcing, and $\sigma_{h}$ is the inter-annual variability in the ice-sheet elevation). Further, they are applied not just over one year as in synchronous coupling, but over the entire period $T_{S} + T_{A}$.

For large ice sheets, this can be modeled as a linearization of the deviation from equilibrium due to a step-wise discontinuous random forcing:

$$\frac{dl}{dt} + l = \frac{\partial^2 \bar{h}(t)}{\partial h^2} h^{(i)}(t) \quad (2)$$

where $l(t)$ are small perturbations in ice-sheet extent, $\bar{h}$ is the ice-sheet time-scale (-7000 year), and $\partial^2 \bar{h}(t)/\partial h^2$ is the dependence of equilibrium ice-sheet size on snowline elevation (-40000). The forcing $h^{(i)}(t)$ is composed of linear segments over time intervals $l_{C_i}$ with random start and end values that depend on $h_{fo}$ and the coupling method. By defining a discrete sequence $l_{C_i}$ and solving Equation (2) over one interval $l_{C_i}$, the sequence of $l_{C}$ is seen to
be a first-order Markov process, and for $t_e \ll \tau$, its variance is (Priestley, 1981; cf. Saltzman and others, 1984):

$$\langle l^2 \rangle = \frac{\partial \text{eq}}{\partial l} \left( \frac{T_s}{\tau} \right) \left( \frac{T_s^2}{2T_s} \right)$$

(3)

For synchronous coupling, $\langle l^2 \rangle = \frac{\partial \text{eq}}{\partial l} = 10^4 m^2$ and $t_e = t_{sto} = 1$ year to represent observed inter-annual variability, which yields $\langle l^2 \rangle \approx 3.4 km$. (The variance of $l(t)$ can be shown to be almost the same as that of $l(t)$ for $t_e = \tau$.) For extrapolated asynchronous coupling, the forcing variance is reduced by a factor $t_{sto}/T_s$ and $t_e$ becomes $T_s + T_A$, so Equation (3) becomes

$$\langle l^2 \rangle = \frac{\partial \text{eq}}{\partial l} \left( \frac{T_s}{\tau} \right) \left( \frac{T_s^2}{2T_s} \right)$$

(4)

For example, with $T_s = 10$ year and $T_A = 990$ year, Equation (4) yields $\langle l^2 \rangle \approx 34 km$.

Our model results are generally consistent with this analysis, as shown for varying $(T_s + T_A)/T_s$ in Figure 4 and for fixed $(T_s + T_A)/T_s$ in Figure 3. In the lower envelope of Figure 4, the rms errors for a variety of long-term forcing functions are about the predicted size and have a roughly square-root dependence on $(T_s + T_A)/T_s$.

For a fixed value of $(T_s + T_A)/T_s$, Figure 5 shows that the magnitude of the error is fairly independent of the individual $T_s$ and $T_A$ values. However, the value of $T_A$ does affect the spectrum of the ice-sheet response by eliminating periods shorter than $-T_A$, as evident in Figure 5b–d.

**Uncertainties**

We have varied most model parameter values over plausible ranges and used other atmospheric forcing patterns, and found only slight (~20%) changes in the asynchronous errors reported here. However, there are other sources of uncertainty as outlined below.

The most appropriate amplitude of the inter-annual forcing $h_{int}$ in Equation (1) is not well constrained by the present data. We have used 100 m, but values as high as 200 m are plausible (cf. Oerlemans, 1979). From Equation (4) and from other runs not shown, the size of the asynchronous errors with extrapolated coupling is proportional to the amplitude of $h_{sto}$, so this source of uncertainty translates to a factor of about 2 in our quantitative results.

The errors are also proportional to $\partial \text{eq}/\partial l$ in Equation (4). This value is relatively small for stable ice sheets as in this study, but it is conceivable that the equilibrium of real Pleistocene ice sheets is extremely sensitive to snowline changes, so the stabilizing feedback in Equation (2) becomes very weak allowing the stochastic forcing to be an important "random-walk" factor in ice-age evolution (e.g. Nicolis, 1982). We have not tuned the model to investigate this regime.

We have implicitly assumed that stochastic ice-sheet forcing is not important for the real ice ages, but is a nuisance to be filtered out during synchronous periods $T_s$. As Hasselmann (1988) points out, if it really is important its statistics should be measured during each synchronous period so that a synthetic stochastic component can be superimposed during the asynchronous periods. In future GCM runs, that (and the spin-up of the upper oceans) would probably require longer synchronous periods than 10 years.

**CONCLUDING REMARKS**

Errors in asynchronous coupling of simple ice-sheet and atmospheric forcing models are due to (i) the phase-lagging of the long-term "orbital" forcing by $-T_A$, and (ii) the amplification and distortion of the stochastic forcing component.

The phase-lag errors can be reduced to negligible levels by using the extrapolated scheme with $T_A$ on the order of 1000 year or less. The remaining errors are then due to the spuriously altered stochastic forcing, which amplifies the resulting ice-sheet fluctuations by a factor of about $(T_s + T_A)/T_s$ compared to synchronous coupling, and concentrates them at periods of $T_A$ and above.

Figure 4 shows the general dependence of error size on $(T_s + T_A)/T_s$ using the extrapolated scheme. The large error spikes such as in Figure 2c have been excluded from Figure 4 on the grounds that they are sporadic, occur only for decaying ice sheets less than ~300 km in extent, and are partly due to inherent model unpredictability. We consider that non-sporadic errors of ~150 km or less will be acceptable for future GCM ice-age simulations, since that is an order of magnitude smaller than the full glacial–interglacial range and is a fraction of the typical horizontal resolution in atmospheric GCMs. To keep asynchronous errors below ~150 km, Figure 4 shows that $(T_s + T_A)/T_s$ must be ~100 or less.

The ratio $(T_s + T_A)/T_s$ is also the approximate increase in efficiency afforded by asynchronous coupling in long-term GCM ice-age simulations, since most of the computer costs will be incurred by the atmospheric and upper oceanic components running only during synchronous periods. Hence the future choice of $(T_s + T_A)/T_s$ will involve a trade-off between efficiency and accuracy.

Coupled atmospheric and oceanic GCMs currently use about 10 supercomputer (CRAY X-MP/48) CPU hours per simulated year, which means that a fully synchronous simulation of the last 150,000 years would require 170 years of CPU time. If asynchronous coupling were used with
\[ \frac{T_g + T_A}{T_A} = 100, \] only 1.7 years of CPU time would be needed. That is still prohibitive, but should become feasible in the 1990s when available computer power increases by one more order of magnitude.

ACKNOWLEDGEMENT

The National Center for Atmospheric Research is sponsored by the U.S. National Science Foundation.

REFERENCES


Pollard and others: Asynchronous coupling of ice-sheet