The important role played by ice on climate is recognized in the ice-albedo feedback effect which is incorporated in most climate models. But the ice under consideration is nearly in equilibrium. But as time goes on the seasonal ice and sea ice, it does not to the permanent ice sheets on earth because they are three-dimensional. A new feature is the ice flow down the ice-sheet slope. This increases the area of the ice-sheet periphery above that produced by local precipitation that is considered in most climate models. As a result the ice-albedo effect will be greater than that given in most climate models. The latter (a positive feedback) is generally smaller than the major negative feedback due to the infrared effect and therefore stable equilibrium is achieved in most climate models. When the positive feedback is thus increased to an amount equal to the infrared negative feedback, stable equilibrium is destroyed and the system changes to natural equilibrium — a mildly unstable form. This may be just what is necessary to explain the advance and retreat of the ice sheets, which cannot be explained by most climate models because in stable equilibrium an ice sheet cannot advance and retreat.

The advance and retreat can be proven to be the manifestation of a neutral equilibrium. At any instant of time in an ice age, including the present time, the climate system is nearly in equilibrium. But as time goes on, the equilibrium point shifts continuously, reflecting the advance of the ice sheet and the cooling of the ocean. Thus an ice age is represented by a continuous sequence of equilibrium points, which, by definition, constitutes a neutral equilibrium. The argument is reinforced by the observation that the "cause" of ice ages is a very small perturbation — the variation of the eccentricity of the earth orbit. According to most climate models, this effect is one order of magnitude too small to generate the climate changes in an ice age. On the other hand in a neutral equilibrium a very weak perturbation can generate slow progressive changes, like the rolling of a cylinder on a plane. As a matter of fact, the small periodic changes of the eccentricity pace the advance and retreat of the ice sheets almost in phase; this is not possible in stable and unstable equilibrium and can be possible only in neutral equilibrium. The empirical conclusion of neutral equilibrium is solidly established. Any theory and any climate model must accommodate this fact.

A deeper understanding requires the pinpointing of the physical origin of the neutral equilibrium. The origin can be shown, though not obviously, to be just the phase equilibrium of water and ice, which is quasi-static and may be considered as neutral equilibrium in a mechanical analogy. The argument is complicated by the fact that the temperature does not remain constant as expected in phase equilibrium but does decrease in an ice age. However, this can be accounted for by the complication of the albedo effect. Hypothetically if ice were brown and without albedo effect, then in an ice age the temperature would not decrease and the situation would be just like ice-water phase equilibrium. The physical origin of neutral equilibrium clarifies the fundamental principle but is not necessary for the mathematical formulation of a dynamic theory of the ice sheets that is given as follows. Instead neutral equilibrium emerges naturally from that theory and the advance and retreat can be explained readily.

Most climate models are static models which cannot explain the advance and retreat — a manifestly dynamic process. A simple but adequate dynamic theory can be formulated considering the ice volume \( V(t) \) and the ocean surface temperature \( T(t) \) as the basic dynamic variables. The equations for their changes can be written down, making use of the latent heat of fusion of ice and the sensible heat capacity of the ocean mixed layer. Albedo positive feedback and infrared negative feedback will be included in the heat balance. The three-dimensional effect of the ice sheet will be introduced in the albedo effect. This involves the geometry of the ice sheet which will be specified in a constraint equation. Then a closed, deterministic set of integral-differential equations may be formulated to describe the dynamics of the ice sheets. The equations may be solved without adjustable parameters. The solutions agree with the observed advance and retreat of ice sheets and the cooling and warming of the ocean. A close examination of the feedbacks involved reveals that the dynamics is indeed like a mechanical system in neutral equilibrium. However, the detailed mathematical theory shows that the neutral equilibrium changes to stable when the ice sheets reach the middle latitudes. They thus stop there in maximum glaciation.

There is no limit on the other end. The northern ice sheet did melt completely. So could the Antarctic ice sheet though it has not in the ice ages. But before the Pleistocene in 99% of the geological time the earth was free of ice sheets. Then, why did the "cause" of ice ages, the eccentricity variations, not generate ice ages in 99% of the earth history? Ice and water can co-exist (in equilibrium) only under unusual conditions, which are not fulfilled generally. But in recent geological times Antarctica drifted to the pole position and high mountains arose on the continents so that permanent ice can survive the summer season and accumulate year after year. Most important, the \( \text{CO}_2 \) content in the atmosphere has been decreasing (Budyko, 1974), cooling down the earth. Eventually the earth was cool enough in Pleistocene to satisfy the neutral equilibrium condition. Then the ice ages began.

But we are now burning fossil fuels to put \( \text{CO}_2 \) back into the atmosphere, going back to pre-Pleistocene conditions. The Antarctic ice sheet will melt away; this will be the principal result of the greenhouse effect. However, current discussion of the greenhouse effect deals with only the warming of the earth and neglects the more serious effect of the melting of the Antarctic ice sheet because climatologists overlooked the effect of ice sheets on climate. This study will change the outlook.

REFERENCE