Simple relations for the close-off depth and age in dry-snow densification

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ABSTRACT. A physical model for the snow/firn densification process (Salamatin and others, 2006) and Martinerie and others’ (1992, 1994) correlation for the firn density at the pore closure are employed to perform a scale analysis and computational experiments in order to deduce simplified relations for the close-off depth and ice age in quasi-stationary ice formation conditions. The critical snow density at which ice-grain rearrangement stops is used to take into account variability of snow structures subjected to densification. The results obtained are validated on a representative set of ice-core data from 22 sites which covers wide ranges of present-day temperatures and ice accumulation rates. A simple analytical approximation for the density–depth profile is proposed.

LIST OF SYMBOLS

\( b \) Accumulation rate in ice equivalent
\( B_h, B_t \) Dimensionless shape factors of the density–depth profile in Equations (3) and (4)
\( C \) Relative slope of the cumulative ice-grain radial distribution function (RDF)
\( g \) Gravity acceleration
\( h \) Depth
\( \rho_1 \) Load pressure
\( Q_p \) Activation energy of dislocation creep
\( R_g \) Gas constant
\( t \) Ice age
\( T \) Temperature (in K)
\( Z \) Coordination number of ice grains
\( \alpha \) Dislocation power creep exponent
\( \gamma \) Densification factor in Equation (7)
\( \zeta \) Fraction of grain surface occupied by excess neck volume due to pressureless sintering
\( \mu \) Non-linear viscosity in power-creep ice-flow law
\( \rho \) Relative density of snow/firn deposits (normalized by \( \rho_1 \))
\( \rho_i \) Density of pure ice
\( \omega \) Densification (compression) rate defined by Equation (1)

Superscripts

\( \tilde{\cdot} \) Modified value
* Reference value
\( \hat{\cdot} \) Scaled (dimensionless) variables

Subscripts

\( c \) Close-off characteristic (firn-to-bubbly-ice transition)
\( s \) Ice-sheet surface
\( 0 \) Critical point (snow-to-firm transition)

INTRODUCTION

The transformation of dry snow into bubbly ice, being a fundamental glaciological phenomenon, is also a key process that links paleoclimatic records of ice properties in glaciers to those of atmospheric gases trapped in the ice (e.g. Schwander, 1989; Barnola and others, 1991; Schwander and others, 1997; Goujon and others, 2003; Blunier and others, 2004). From this point of view, the most important general characteristics of the snow/firn densification process are the age \( t_c \) of ice at the pore closure and the close-off depth \( h_c \) at which all pores become closed and firn transforms to bubbly ice with the close-off relative density \( \rho_c \).

As a continuation of previous studies (Arnaud and others, 1998, 2000), an improved physical model for the snow/firn densification on the ice-sheet surface has recently been developed by Salamatin and others (2006). It has been further constrained and validated by available data (Salamatin and others, in press). Based on Alley’s (1987) and Arzt’s (1982) theories, the model considers the overall vertical (uniaxial) compression of the snow and firn under increasing overburden pressure as a sum of two constituents, one caused by rearrangement of ice grains as rigid particles and another controlled by grain plasticity. In contrast to previous studies, it also takes into account the dilatancy effects in the ice particle repacking. As a result, the first (snow) stage of densification, being dominated by the ice particle rearrangement, is simultaneously influenced by a gradual increase in the dislocation creep of grains. By definition (Arnaud and others, 1998, 2000), the second (firm) stage starts when the grain rearrangement ceases at the closest (dense) packing of ice crystals. Following Arzt (1982), the initial firm structure is described (Salamatin and others, 2006, in press) by the critical coordination number \( Z_0 \sim 6.5–8.0 \) and by the slope of the cumulative ice-particle radial distribution function (RDF) \( C \sim 40–60 \). These microstructural parameters determine the critical relative density at the snow-to-firm transition \( \rho_0 \sim 0.7–0.75 \). Traditionally, the boundary between the two densification stages is assumed, after Anderson and Benson (1963), at a considerably lower relative density of 0.6 corresponding to the specific bend observed in many ice-core density profiles. Modeling by Salamatin and others (2006, in press) has confirmed the earlier finding by Ebinuma and co-workers (Ebinuma and others, 1985; Ebinuma and Maeno, 1985, 1987) that this first sharp decrease in the densification rate manifests only the onset of an intermediate regime, in which particle rearrangement and plasticity work together. The
dislocation creep takes over, and the firm stage begins at the higher critical relative densities.

Thus, the critical density \( \rho_0 \) becomes one of the principal microstructural parameters which control the snow/firn densification in modeling approaches (Arnaud and others, 1998, 2000; Salamatin and others, 2006, in press). The initial (surface) snow build-up and the evolution of the snow/firn structure with depth depend on ice formation conditions (Alley, 1988). Ice-core data analysis performed by Salamatin and others (in press) shows that higher critical densities generally correspond to higher temperatures \( T \) and higher surface snow densities \( \rho_s \), although without clear quantitative correlation. Similar observations were earlier reported by Benson (1962) and Arnaud (1997), but the definition of the critical density was different. It was suggested that meteorological conditions such as wind speed, surface temperature, temperature gradients and insolation (e.g. Craven and Allison, 1998; Lipenkov and others, 1998; Bender, 2002; Raynaud and others, 2007) and, possibly, precipitation processes can affect the properties of the near-surface snow and, thus, the densification of snow/firn strata. As summarized in Table 1, two types of snow microstructures (L and H groups of ice cores) can be roughly distinguished on the basis of model constraining (Salamatin and others, 2006) predict a certain similarity between different profiles of the relative density \( \rho \) vs depth \( h \). The critical relative density \( \rho_0 \) appears in the above-cited papers in the constitutive relations for the macroscopic snow/firn compression rate \( \omega \) as the typical scale of \( \rho \), being close to the mean value of the relative density over the surface layer above the close-off level \( h_c \).

The vertical velocity of a reference snow/firn particle is \( b/\rho \), and, by definition,

\[
\frac{1}{\rho} \frac{d\rho}{dh} = \frac{3 \omega \rho_0^2}{B},
\]

where in a general form (Arnaud and others, 2000; Salamatin and others, 2006)

\[
\omega = \left( \frac{p}{\rho} \right)^{\alpha} \mu(T) \left( \frac{\rho}{\rho_0} \right)^2 f \left( \frac{\rho}{\rho_0} \right).
\]

Here \( \alpha \) is the creep index, \( \mu \) is the Arrhenius-type temperature-dependent coefficient of non-linear viscosity in the ice-flow law, \( f \) is a function of \( \rho/\rho_0 \), temperature, and microstructural parameters, and \( \rho_0 \) is the load pressure calculated as \( \rho_0 = g \rho \int_0^h \rho \, dh \), where \( g \) is the gravity acceleration.

<table>
<thead>
<tr>
<th>Table 1. Snow/firn densification parameters</th>
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<tbody>
<tr>
<td>Parameter</td>
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<tr>
<td>Environmental ice-formation conditions</td>
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<td>Surface snow relative density</td>
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<td>Surface temperature (°C)</td>
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<td>Microstructural characteristics</td>
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<tr>
<td>Critical coordination number</td>
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<td>RDF slope</td>
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<td>Critical relative density</td>
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<td>Critical bonding factor</td>
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<td>Rheological parameters</td>
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<td>Creep index</td>
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<td>Non-linear viscosity of ice (^1) (MPa·s (^a))</td>
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<tr>
<td>Activation energy of dislocation creep (kJ mol(^{-1}))</td>
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<tr>
<td>Mean shape factors of density-depth profiles</td>
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<td>Close-off depth factor</td>
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<td>Close-off ice-age factor</td>
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\(^1\)At the reference temperature of \( T^* = 215.7 \) K.
After substitution of the above expressions for \( \omega \) and \( \rho_c \) in terms of scaled variables \( \bar{\rho} = \rho/\rho_0 \) and \( \bar{h} = h/h_c \), Equation (1) transforms to

\[
\frac{d\bar{\rho}}{dh} = \frac{3\rho_0 (g\rho_0)^\alpha h_c^{\alpha+1}}{b\mu} \left( \frac{1}{\bar{\rho}} \int_0^{\bar{h}} \bar{\rho} \, dh \right)^\alpha.
\]  (2)

Integration with respect to \( \bar{h} \) from 0 to 1 and with respect to \( \bar{\rho} \) from \( \rho_c/\rho_0 \) (\( \rho_c \) is the relative density of surface snow) to \( \rho_c = \rho_c/\rho_0 \) yields

\[
h_c = B_h \left[ \frac{b\mu}{(g\rho_0)^\alpha \rho_0} \right]^{\frac{1}{\alpha}}.
\]  (3)

where

\[
B_h(\rho_0, \rho_c) = \left[ \frac{1}{3} \int_0^{\rho_c} f(\bar{\rho}) \, d\rho_0 \right]^{\frac{1}{\alpha}} \left[ \frac{1}{3} \int_0^{\rho_c} \left( \frac{1}{\bar{\rho}} \int_0^{\bar{h}} \bar{\rho} \, dh \right)^{\alpha} \, d\rho_0 \right]^{\frac{1}{\alpha}}.
\]

Accordingly, if \( \rho_0 \) represents the mean density of the snow/firn layer above the close-off level then the ice age at pore closure \( t_c = \rho_0 h_c/b \), and Equation (3) can be rewritten as

\[
t_c = B_t \left[ \frac{\mu \rho_0}{(g\rho_0)^\alpha} \right]^{\frac{1}{\alpha}}.
\]  (4)

This defines the \( B_t \) factor and assumes that \( B_t \approx B_h \).

Based on the scale analysis, Equations (3) and (4) explicitly reveal the principal intrinsic links between the close-off characteristics \( (h_c, t_c) \), snow/firn rheological properties \( (\alpha, \mu) \) and climatic conditions \( (b, T) \). As a consequence, by definition, the shape factors of density–depth profiles \( B_i \) and \( B_h \) are expected to be constant or, at least, depend only on structural characteristics (e.g. \( \rho_0, \rho_c \)). Relations (3) and (4) were envisaged by Salamatin and others (in press). However, the coefficients \( B_h \) and \( B_t \) were introduced formally and estimated directly on the basis of ice-core data. They had different values for each of the two established L and H types of snow microstructures with noticeable (\( \pm 5\%\)–10\%) variations (see Table 1). In the following section, we use the snow/firn densification model to specify the \( B_i \) and \( B_h \) relationships.

### COMPUTATIONAL EXPERIMENTS AND ICE-CORE DATA

The physical snow/firn densification model (Salamatin and others, 2006) was constrained and validated on a representative set of ice-core density measurements at 22 sites in the Antarctic and Greenland ice sheets with wide range of present-day temperatures from \(-57.5\) to \(-10^\circ\)C and ice accumulation rates from 2.2 to 330 cm a\(^{-1}\). The model parameters (i.e. the factor \( f(\bar{\rho}) \) in Equation (2)) were tuned so as to fit the simulated density–depth curves to the experimental data. Here we describe a series of computations performed with the recommended (mean) parameters from Table 1. We study the analytical expressions of the \( B \) factors in Equations (3) and (4) for different snow/firn structures, characterized by the critical densities \( \rho_0 \) and possible variations of the close-off densities \( \rho_c \).

Based on Martinerie and others (1992, 1994), a linear empirical correlation between the close-off relative density \( \rho_c \) and the firm temperature \( T \) (in K),

\[
\rho_c = 0.9 - 5.39 \times 10^{-4} (T - 235),
\]  (5)

can be employed after Lipenkov and others (1999) to predict the close-off depth \( h_c \) and ice age \( t_c \) from the model and to calculate the \( B_h \) and \( B_t \) values in Equations (3) and (4). Additionally, for each temperature, maximum deviations of \( \pm 0.01 \) from \( \rho_c \) given by Equation (5) are also tested.

In full agreement with the scale analysis, the calculations confirm that \( B_i \) and \( B_t \) do not directly depend on \( T \) and do not differ from each other by more than \( \pm 2\%\) on average. Accordingly, the 22 best-fit ratios \( B_i/B_t \) inferred in Salamatin and others (in press) for Martinerie’s relation (5) are equal to 1 within the standard deviation of 1\%.

The computational experiments at the creep exponent \( \alpha = 3.5 \) reveal that a power approximation \( f \sim (\bar{\rho})^{2.5+1} \) can be assumed. Hence, the integral of \( f(\bar{\rho}) \) in \( B_h \) coefficient in Equation (3) is proportional to \( (\rho_c/\rho_0)^{2.5+2} - (\rho_c/\rho_0)^{2.5+2} \), where the second term appears to be negligibly small. As a result, \( B_h \) does not depend on \( \rho_c \) and is found to be inversely proportional to \( \rho_0 \). Finally, the proportionality between \( B_h \) and \( \rho_c \) can be established directly from the simulations:

\[
B_h \approx B_t \approx 2.32 \rho_0^2 / \rho_c^2.
\]  (6)

The analytical accuracy of this approximation is not worse than \( \pm 1\% \) for \( B_t \) and \( \pm 3\% \) for \( B_h \). The best-fit estimates of the products \( B_h \rho_0 \rho_c^2 / \rho_c^5 \) and \( B_t \rho_0 \rho_c^2 / \rho_c^5 \) obtained in Salamatin and others (in press) are plotted against temperature \( T \) in Figure 1 by solid and open circles, respectively. The solid line in the figure corresponds to Equation (6) and practically coincides with the mean-square approximation (dashed line) of the observational data. The relative standard deviation does not exceed 2\%. It is partly caused by local changes in snow/firn structures, i.e. in microstructural parameters \( Z_0 \), \( C \), and largely by deviation of \( \zeta_0 \) from its mean recommended value 0.55 (see Table 1).

Thus, Equations (3), (4) and (6) consistently, within a few percent, predict the general close-off characteristics of the firm-to-bubbly-ice transition at given (present-day) climatic conditions \( b, T \) provided that the critical density \( \rho_0 \) and close-off density \( \rho_c \) of the snow/firn structure are known. The latter parameters, although rather stable, are primarily influenced by temperature and other meteorological conditions (Martinerie and others, 1992, 1994; Arnaud, 1997; Arnaud and others, 1998; Lipenkov and others, 1999; Raynaud and others, 2007; Salamatin and others, in press). \( \rho_0 \) can be estimated on the basis of the data presented in Table 1, while \( \rho_c \) is conventionally determined by Equation (5).

**Fig. 1.** The best-fit estimates of the products \( B_h \rho_0 \rho_c^2 / \rho_c^5 \) and \( B_t \rho_0 \rho_c^2 / \rho_c^5 \) vs temperature \( T \) (solid and open circles) deduced by Salamatin and others (in press) from the 22 ice-core density profiles over the Antarctic and Greenland ice sheets at \( \rho_0 \) given by Equation (5) as compared to Equation (6) (solid line) and the mean-square approximation (dashed line).
Assuming that for a certain site under consideration the snow/firn structure development and the critical density do not change significantly with the climate, we can rewrite Equations (3–6) in terms of relative variations of the close-off characteristics for two different stationary conditions:

\[
\frac{h_c}{h_0} = \left(\frac{b'}{b}\right)^{\frac{1-c}{1-c_0}} \exp\left[\frac{Q_p'}{R_h (1 + \alpha)} \left(\frac{1}{T} - \frac{1}{T_c}\right)\right],
\]

\[
\frac{t_c}{t_0} = \left(\frac{b'}{b}\right)^{\frac{1-c}{1-c_0}} \exp\left[\frac{Q_p'}{R_h (1 + \alpha)} \left(\frac{1}{T} - \frac{1}{T_c}\right)\right].
\]

Here \(R_h = 8.314\) (mol K\(^{-1}\)) is the gas constant, and \(h_c\) and \(t_c\) are the close-off depth and ice age determined (measured) at the reference (present-day) ice accumulation rate \(b'\) and surface (firm) temperature \(T'\). The apparent activation energy \(Q_p' \approx 6.36\) kJ mol\(^{-1}\) is a modification of \(Q_p\) from Table 1 additionally corrected to take into account the dependence of Equation (5) of \(p_c\) on temperature in Equation (6). This form of Equations (3) and (4) may be especially useful in paleo-reconstructions and sensitivity studies.

**DENSITY–DEPTH PROFILE**

A simple analytical approximation of the density–depth profile for the snow/firn layer can be useful in applications. Based on ice-core measurements (Lipenkov and others, 1997) at Vostok station, Antarctica, an exponential presentation was proposed by Salamatin and others (1997) and confirmed by Ekaykin and others (2003). Subject to the condition that the critical relative density \(p_0\) equals the mean relative density of the snow/firn layer above the close-off level \(h_c\), we can write this relationship as

\[
\rho = 1 - \frac{\gamma(1-p_0)}{1 - e^{-\gamma h/h_c}},
\]

where the densification factor \(\gamma\) is expressed via \(p_c\) and \(p_0\) by the following equation

\[
\gamma e^{-\gamma} = \frac{1 - p_c}{1 - p_0},
\]

which is a consequence of Equation (7) at \(h = h_c\), where \(\rho = p_c\). The parameter \(\gamma\) as a function of the righthand side of Equation (8) is plotted in Figure 2.

To illustrate the applicability of Equations (7) and (8) in combination with Equations (3–6), two limiting cases of Antarctic ice cores from Vostok station (Lipenkov and others, 1997) and H72 site (Nishio and others, 2002) are considered as typical respective representatives of L and H groups. Present-day climatic conditions at these sites and close-off characteristics calculated from Equations (3–6) for the critical densities deduced in Salamatin and others (in press) are presented in Table 2. As expected, \(h_c\) and \(t_c\) do not differ by more than 2% from the corresponding best-fit estimates given in parentheses. However, the accuracy reduces to 3.5–5% if the mean critical densities for L and H structures from Table 1 are used in calculations. The relative density–depth profiles described by Equations (7) and (8) are compared with the observational data in Figure 3. These exponential curves predict the general course of the densification process quite well, but do not catch the initial depositional and/or diagenetic phase of the snow metamorphism (Alley, 1988) within a few (3–5) uppermost meters. In the case of the L group, under cold and low-wind conditions affected by insolation, a low-density firn microstructure is formed at decreased rates of the near-surface densification (see Fig. 3a). On the other side, the H group is characterized by the intense pressureless sintering in the near-surface snow layer, resulting in a high-density firn microstructure (see Fig. 3b).

**CONCLUSION**

Simple relationships (3), (4) and (6) for the depth \(h_c\) and ice age \(t_c\) of the firm-pore closure are derived on the basis of the general snow/firn densification model (Salamatin and others, 2006). Together with Martinerie and others’ (1992, 1994) correlation (5) for the close-off relative density \(p_c\), they allow an approximate, though fairly accurate (within a few percent), description of the densification process in quasi-stationary climatic conditions \((b, T)\) with the firm structure specified by the critical relative density \(p_0\). The importance of this microstructural parameter was earlier emphasized by...
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Table 2. Climatic conditions and close-off characteristics at Vostok and H72 sites

<table>
<thead>
<tr>
<th>Site</th>
<th>Group</th>
<th>$T$</th>
<th>$b$</th>
<th>$\rho_0$</th>
<th>$h_c$</th>
<th>$\tau_c$</th>
<th>$\rho_c$</th>
<th>$\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vostok</td>
<td>L</td>
<td>215.7</td>
<td>2.15</td>
<td>0.714$^a$</td>
<td>99.2 (97.5$^a$)</td>
<td>3.29 (3.17$^a$)</td>
<td>0.91</td>
<td>2.0</td>
</tr>
<tr>
<td>H72</td>
<td>H</td>
<td>253.9</td>
<td>3.45</td>
<td>0.736$^a$</td>
<td>54.1 (54.9$^a$)</td>
<td>0.115 (0.116$^a$)</td>
<td>0.89</td>
<td>1.55</td>
</tr>
</tbody>
</table>

$^a$The best-fit estimates from Salamatin and others (in press).

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REFERENCES


