Parameter sensitivity of a distributed enhanced temperature-index melt model

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ABSTRACT. We investigate the sensitivity of a distributed enhanced temperature-index (ETI) melt model, in order to understand which parameters have the largest influence on model outputs and thus need to be accurately known. We use melt and meteorological data from two Alpine glaciers and one glacier in the Andes of Chile. Sensitivity analysis is conducted in a systematic way in terms of parameters and the different conditions (day, night, clear-sky, overcast), melt seasons and glaciers examined. The sensitivity of total melt to changes in individual parameters is calculated using a local method around the optimal value of the parameters. We verify that the parameters are optimal at the distributed scale and assess the model uncertainty induced by uncertainty in the parameters using a Monte Carlo technique. Model sensitivity to parameters is consistent across melt seasons, glaciers, different conditions and the daily statistics examined. The parameters to which the model is most sensitive are the shortwave-radiation factor, the temperature lapse rate for extrapolation of air temperature, the albedo parameters, the temperature threshold and the cloud transmittance factor parameters. A parameter uncertainty of 3% results in a model uncertainty of 5.6% of mean melt on Haut Glacier d’Arolla, Switzerland.

1. INTRODUCTION

Predictions of worldwide changes in glaciers in response to a changing climate rely on the use of models of glacier mass balance, i.e. glacier accumulation and ablation, as well as models describing the dynamics of ice flow. Ablation models of different complexity have been used to estimate glacier melt and mass balance, and for future predictions in particular (e.g. Huss and others, 2008; Hock and others, 2009; Farinotti and others, 2012; Immerzeel and others, 2012). For example, Nolin and others (2010) and Hock and others (2009) used a degree-day model, Huss and others (2008) and Farinotti and others (2012) employed a temperature-index model, including a shortwave radiation index, to simulate the response of Swiss glaciers to a changing climate and Le Meur and others (2007) used an energy-balance model for future simulations of the mass balance of an Alpine glacier.

For regional estimates of glacier changes (e.g. Hock and others, 2009), as well as for applications in data-scarce regions (Ragettli and Pellicciotti, 2012), melt models often need to be applied with little or no recalibration of model parameters. Even in the European Alps, applications outside the few well-studied glaciers rely on limited datasets that do not allow optimal calibration of all the parameters of a given model. The data most commonly available for model calibration and validation are glacier runoff (e.g. Huss and others, 2008) and mass-balance data (e.g. Finger and others, 2011). The former, especially, does not allow internal validation of the model by testing that the single processes are correctly reproduced (Pellicciotti and others, 2012), and might result in more than one set of parameter values providing the same model performance; this has been referred to as an ‘equifinality problem’ (Beven and Freer, 2001; Wagener and others, 2003; Beven, 2006).

In view of this limitation, there is a strong need to identify which parameters melt models are sensitive to, and which could be discarded from the calibration procedure because their impact on model output is small (Saltelli and others, 2000). This is particularly important for empirical models that rely more or less heavily on empirical parameters, such as the simple degree-day model and, to a lesser extent, enhanced versions of this approach (Hock, 1999; Pellicciotti and others, 2005). However, it is also relevant for more physically based approaches, such as energy-balance models, since when models are applied at the distributed scale, numerous additional parameters are introduced to extrapolate meteorological and surface variables (MacDougall and Flowers, 2011).

Sensitivity analysis is a well-established technique in hydrological modelling (e.g. McCuen, 1973; Saltelli and others, 2000; Kunsmann and others, 2006; Tang and others, 2007) which serves several purposes. One of these is to identify the most influential parameters on a given metric of model performance or output (McCuen, 1973; Saltelli and others, 2000). Less common is its use in studies of glacier mass balance and runoff, but some examples do exist (e.g. Klok and Oerlemans, 2002; Hock and Holmgren, 2005; Pellicciotti and others, 2005; Anslow and others, 2008; MacDougall and Flowers, 2011; MacDougall and others, 2011; Fitzgerald and others, 2012), all of which include analysis of model sensitivity to parameters in the melt modelling. However, most of these studies consider the model sensitivity to only one or a few parameters, often those taken from the literature or where the estimates were known to be less accurate. To date, only MacDougall and Flowers (2011), MacDougall and others (2011) and Fitzgerald and others (2012) have analysed the sensitivity of a distributed melt or mass-balance model to single variations in all of its parameters.

Several sensitivity methods exist. A main difference is between local and global approaches. In local methods, normally associated with so-called ‘one factor at a time’ (OAT) changes in parameters, the local response of the outputs, obtained by varying the factors one at a time, is investigated, while holding all other factors fixed to a central (optimal) value (Saltelli and others, 1999). In global
1. To identify which parameters most affect the ETI model outputs, i.e. which parameters the model is most sensitive to, and

2. To quantify the uncertainty associated with small variations in the parameters.

Several authors have indicated that sensitivity depends on the period and case study considered (e.g. McCuen, 1973). MacDougall and Flowers (2011) showed that the sensitivity of a distributed energy-balance model was different for two neighbouring glaciers. For this reason, we apply the sensitivity analysis to different modelling periods and glaciers, in order to test the robustness of our results and avoid conclusions that are time- or site-specific.

We investigate the parameter sensitivity in a systematic manner in terms of the number of parameters changed, the different conditions (day, night, clear-sky, overcast) during one season on one glacier, different melt seasons and glaciers. We use meteorological and ablation datasets from two Alpine glaciers, Haut Glacier d’Arolla (ablation seasons 2001, 2006) and Gornergletscher (ablation season 2006), and from Glaciar Juncal Norte (ablation season 2008/09) located in the semi-arid Andes of central Chile. As a first step we analyse parameter sensitivity according to an OAT method for different seasons and glaciers to reveal the most sensitive parameters (aim 1 above). For Haut Glacier d’Arolla in 2001 we also check the ranking of parameter sensitivity changes under different conditions (day, night, clear-sky, overcast) in one melt season. In a second step, we investigate the optimality of the parameters at the distributed scale and the model uncertainty corresponding to a fixed amount of uncertainty in model parameters (aim 2) for Haut Glacier d’Arolla during the 2001 ablation season.

2. STUDY SITES AND DATA

2.1. Study sites

Haut Glacier d’Arolla is situated at the head of the Val d’Hérens, Valais, Switzerland. The glacier is ~4 km long, has an area of ~6.3 km² and ranges in elevation from 2560 to 3540 m a.s.l. (Fig. 1). It consists of an upper basin with northwesterly aspect feeding a northward-flowing glacier tongue (Pellicciotti and others, 2005).

Gornergletscher is located in Valais, at the head of the Mattertal. It is a polythermal glacier with areas of cold ice at temperatures below the pressure-melting point. It has a length of 14 km, covers a total area of 57.5 km² and ranges in elevation from 2000 to 4600 m a.s.l. The glacier tongue...
covers an elevation range from 2600 to 2300 m a.s.l. Both sides of the tongue are covered with debris (Müller, 2010).

Glacier Juncal Norte is located in the upper Aconcagua river basin, in the semi-arid Andes of central Chile. The glacier is ~7.5 km long, has an area of ~7.6 km² and ranges in elevation from 2900 to 6100 m a.s.l. (Pellicciotti and others, 2008). The semi-arid Andes of central Chile are characterized by a pronounced seasonality, with runoff in summer (December–February) originating mainly from snow and glacier melt (Peña and Nazarala, 1987; Masiokas and others, 2006), and precipitation largely absent (Pellicciotti and others, 2008). Most of the annual precipitation occurs in winter (June–August).

2.2. Meteorological and ablation measurements

The meteorological and ablation datasets collected on Haut Glacier d’Arolla during the 2001 and 2006 ablation seasons (referred to as HGdA01 and HGdA06, respectively) are described in more detail by Strasser and others (2004), Pellicciotti and others (2005) and Carenzo and others (2009). Here we use data from two automatic weather stations (AWSs) on- and off-glacier (Fig. 1). AWS 1 was installed on the glacier in the two ablation seasons and the proglacial station recorded the same variables as on Haut Glacier d’Arolla, while the proglacial station recorded temperature and precipitation. (See Carenzo and others, 2009, and Müller, 2010, for details of sensors, set-up and recorded variables.)

For the 2008/09 ablation season on Glacier Juncal Norte (referred to as JNG08/09), two stations, AWS 1 and AWS 3, were installed on the tongue of the glacier and in the proglacial valley, respectively. The stations recorded the same meteorological variables as those on Haut Glacier d’Arolla (Ragettli and Pellicciotti, 2012). (Details of the set-up and measurements are provided by Pellicciotti and others, 2008, and Ragettli and Pellicciotti, 2012.)

The measurement periods and coordinates of all AWSs are given in Table 1. For each case study, data from the on-glacier AWS are used to force the model. The off-glacier AWS provides data for calculation of the cloud factor and precipitation measurements.

2.3. Initial snow depth

Initial conditions of snow depth are required to run the model simulations. Initial snow depth was estimated for every gridcell of the watershed on all three glaciers, by linear extrapolation of snow depth measurements taken on each glacier at the beginning of the observation period. On Gornergletscher and Glaciar Juncal Norte, measurements were mostly carried out on the lower sections of the glacier, because of restricted access (see Müller, 2010; Ragettli and Pellicciotti, 2012, for the locations). The initial snow depth observations were interpolated using an elevation gradient and corrected with a residual field (based on the inverse distance method) to obtain a map of gridded values of initial snow water equivalent (Müller, 2010). If the slope of a watershed cell was >45°, initial snow depth was set to zero. Snow depths were converted to snow water equivalent (SWE) by means of measured density. Density was measured on each glacier at the time of the snow depth survey at one or two locations. (Details of the measurements are given by Pellicciotti and others, 2005; Carenzo and others, 2009; Müller, 2010; Ragettli and Pellicciotti, 2012.) Because of the simple extrapolation used and the discrete nature of the observations of snow depth and density, the initial distribution of SWE is subject to uncertainty. This might influence the model performance and its ability to reproduce the

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**Table 1. Measurement periods, modelling periods and coordinates of the AWSs for all glaciers and melt seasons. In parentheses after the measurement period are periods of data gaps.**

<table>
<thead>
<tr>
<th>Station</th>
<th>Location</th>
<th>Elevation m a.s.l.</th>
<th>Latitude</th>
<th>Longitude</th>
<th>Measurement period</th>
</tr>
</thead>
<tbody>
<tr>
<td>AWS 0 Proglacial</td>
<td>2507</td>
<td>45°59′24.9″N</td>
<td>7°30′23.00″W</td>
<td>since Nov 2000</td>
<td></td>
</tr>
<tr>
<td>AWS 1 Glacier</td>
<td>2830</td>
<td>45°58′24.95″N</td>
<td>7°31′24.70″W</td>
<td>30 May–11 Sep 2001 (21 Jun–18 Jul)</td>
<td></td>
</tr>
<tr>
<td>AWS 1 Glacier</td>
<td>2830</td>
<td>45°58′24.95″N</td>
<td>7°31′24.70″W</td>
<td>26 May–10 Oct 2006</td>
<td></td>
</tr>
<tr>
<td>Gornergletscher 2006 (modelling period: 26 Apr–11 Sep 2006)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AWS 1 Proglacial</td>
<td>2648</td>
<td>45°58′37.05″N</td>
<td>7°48′06.61″W</td>
<td>25 May–11 Sep 2006</td>
<td></td>
</tr>
<tr>
<td>AWS 2 Glacier</td>
<td>2604</td>
<td>45°57′38.77″N</td>
<td>7°48′01.58″W</td>
<td>26 Apr–11 Sep 2006 (11–23 May, 26–31 Aug)</td>
<td></td>
</tr>
<tr>
<td>AWS 2 Proglacial</td>
<td>2811</td>
<td>32°58′27.64″S</td>
<td>70°06′40.81″W</td>
<td>7 Dec 2008 to 15 Mar 2009</td>
<td></td>
</tr>
<tr>
<td>AWS 1 Glacier</td>
<td>3127</td>
<td>32°59′26.58″S</td>
<td>70°06′31.27″W</td>
<td>7 Dec 2008 to 15 Mar 2009</td>
<td></td>
</tr>
</tbody>
</table>
evolution of the snowpack. Figure 2 shows the initial SWE distribution per elevation band (bandwidth 100 m) for every glacier and melt season. On Haut Glacier d’Arolla there was considerably more snow at the beginning of the season in 2001 than in 2006. For both GG06 and JNG08/09 there was less snow at the beginning of the simulation and it was mainly located at higher altitudes, where temperatures are lower and the snow is unlikely to melt (Ragettli and Pellicciotti, 2012).

3. ABLATION MODEL

In the distributed ETI model, melt rate, $M$ (mm w.e. h$^{-1}$), is computed as (Pellicciotti and others, 2005, 2008; Ragettli and Pellicciotti, 2012):

$$ M = \begin{cases} T_F \cdot T + SRF \cdot (1 - \alpha) \cdot I & \text{if } T > T_{th} \\ 0 & \text{if } T \leq T_{th} \end{cases} $$

where $T$ is the hourly mean air temperature at the screen level ($^\circ C$), $\alpha$ is the albedo and $I$ is the incoming shortwave radiation (W m$^{-2}$). The temperature factor, $T_F$ (mm h$^{-1}$ °C$^{-1}$), and shortwave-radiation factor, $SRF$ (m$^2$ mm W$^{-1}$ h$^{-1}$), are empirical parameters. $T_{th}$ is a threshold temperature above which melt is assumed to occur.

Incoming shortwave radiation in every gridcell is computed using a non-parametric clear-sky model based on that of Iqbal (1983) and described in detail by Pellicciotti and others (2011). Terrain parameters and the solar position, as well as the interaction between solar radiation and the topography, are derived using the vectorial algebra approach of Corripio (2003). The only parameters needed in the clear-sky solar radiation model are the visibility, $vis$, and ozone layer thickness, $O_3$. The net effect of clouds is modelled using cloud transmittance factors, $ctf$, computed as a function of the diurnal temperature range, following Pellicciotti and others (2011). Use of a daily cloud factor to account for the effect of clouds on the clear-sky global irradiance is a widely used approach in mesoscale atmospheric studies (e.g. Thornton and Running, 1999; Pfister and others, 2003; Fitzpatrick and others, 2004) and it has recently been used in distributed models of glacier melt (Klok and Oerlemans, 2002; Anslow and others, 2008; Ragettli and Pellicciotti, 2012). The approach is based on the relationship between daily cloud transmittance, $ctf$, and the daily temperature range:

$$ ctf = cf_1 + cf_2 \cdot \Delta T, $$

where $\Delta T$ is the daily temperature range measured at an off-glacier station and $cf_1$ and $cf_2$ are empirical coefficients (Pellicciotti and others, 2011). To avoid underestimating the clear-sky days radiation, a cloud factor threshold, $ctf_{th}$ is used, following Pellicciotti and others (2011). Cloud factor values greater than or equal to this threshold are set to 1, which corresponds to clear-sky conditions.

The albedo of ice, $\alpha_{ice}$, and of the surrounding bare terrain, $\alpha_{bare}$ (the latter used in the solar radiation model; Pellicciotti and others, 2011) are assumed to be constant. Daily snow albedo is calculated using two different parameterizations: (1) the US Army Corps of Engineers (1956) approach and (2) the parameterization of Brock and others (2000). In the first parameterization, snow albedo, $\alpha_s$, is assumed to decay exponentially with the number of days, $n$, since the last significant snowfall, $ss$:

$$ \alpha_s = \alpha_{1us} + \alpha_{2us} \cdot e^{(k_{pos, reg} \cdot n)}, $$

where $\alpha_{1us}$, $\alpha_{2us}$ and $k_{pos, reg}$ are empirical parameters. $k$ differs for positive and negative temperatures.

According to Brock and others (2000), daily snow albedo is computed as a logarithmic function of the accumulated daily maximum positive air temperature, $T_{max}$ ($^\circ C$), since the last significant snowfall, $ss$:

$$ \alpha_s = \alpha_{1br} - \alpha_{2br} \cdot \log_{10}(T_{max}), $$

where $\alpha_{1br}$ and $\alpha_{2br}$ are empirical parameters.

Temperature and precipitation measured at the point scale of the AWSs are extrapolated to the gridcells of the watershed with a temperature lapse rate, $LR$ (°C m$^{-1}$), that is constant in time, and uniform in space and precipitation gradient, $\Gamma_p$ (mm m$^{-1}$). To distinguish between solid and liquid precipitation a simple phase threshold, $p_{th}$ (°C), is used.

In addition to the melt parameters for temperate, debris-free ice (SRF and TF), separate values for cold ice and debris-covered ice for the ETI model were introduced by Müller (2010) for GG06, to take into account the different surface characteristics of Gornergletscher. These are: $SRF_{coldice}$, $SRF_{debrisice}$, $TF_{coldice}$, $TF_{debrisice}$ and are also considered in the sensitivity analysis.

4. SENSITIVITY TO INDIVIDUAL PARAMETERS

The approach used to compute the sensitivity to individual parameters is an OAT method, adopted from Anslow and others (2008) and Ragettli and Pellicciotti (2012). The method was also used by MacDougall and Flowers (2011). This is a well-established method for calculation of model sensitivity (e.g. McCuen, 1973; Saltelli and others, 1999). We varied the parameters about the optimal value in 5% increments from $-20\%$ to $+20\%$. We use a larger range than commonly adopted to investigate the linearity of the model response. In each model run, one parameter was changed while the others remained fixed at their optimal value. For every parameter change, the modelled total melt (cumulative seasonal melt occurring over the whole watershed) was computed as a percentage of the total melt modelled with the optimal value. Sensitivity was calculated by fitting a second-order polynomial to these results for a given parameter and then determining the slope of the polynomial about the optimal value (Fig. 3). (Details of the methods are given by Ragettli and Pellicciotti, 2012.) The optimal parameter values were calibrated in ad hoc studies at the...
point scale, except for the extrapolation parameters. Their sources are listed in Table 2.

The analysis was performed separately for total melt over the entire basin, on the glacier and on the non-glacierized slopes. We also considered, separately, sensitivity for the daily minimum melt, daily mean melt, daily maximum melt and daily standard deviation of melt, spatially averaged over the whole watershed and temporally averaged over the number of modelling days. These statistics are referred to here as ‘daily min’, ‘daily mean’, ‘daily max’ and ‘daily stdv’, respectively. A positive sensitivity denotes that an increase in the parameter causes an increase in total melt; a negative sensitivity that an increase in the parameter results in a reduction in melt. All model runs were conducted using the US Army Corps of Engineers (1956) approach for albedo, except for the separate, specific runs to assess the model sensitivity to the parameters \(\alpha_{1\text{br}}\) and \(\alpha_{2\text{br}}\) of the Brock and others (2000) parameterization.

The method used implies that the initial parameter values are the optimal ones, otherwise sensitivity is meaningless, as it could be conducted in any region of the parameter space with little relevance in terms of model performance. Parameter optimization depends, however, on the period considered, optimization methods and type of input data used to force the models (measured or simulated) and validation datasets employed to define the model performance, as well as the spatial scale of the model application. Parameters optimized at the point scale, as by Pellicciotti and others (2005) or Carenzo and others (2009), might not be the optimal ones for distributed applications of the model. The assumption that the optimal parameters (calibrated at the point scale) used in this work are also the optimal values at the distributed scale is considered in Section 5.

In general, the parameter values for Haut Glacier d’Arolla are likely to be optimal also at the distributed scale of this application, because Haut Glacier d’Arolla is a small glacier where variability in surface and meteorological conditions can be expected to be small. The glacier has also been extensively investigated (e.g. Brock and others, 2000; Pellicciotti and others, 2005, 2011; Carenzo and others, 2009), so quite a large dataset of observations exists for model calibration. For the two other glaciers, more uncertainty is to be expected, given the more limited

![Table 2. Optimal parameters for the sensitivity analysis to individual parameter changes](image)
datasets used to optimize the model parameters and the glaciers’ larger elevation ranges and types of surface characteristics.

For JNG08/09 the optimal value of TF is zero (Ragettli and Pellicciotti, 2012), therefore this parameter is varied from 0 to 0.016, which corresponds to the range over which it is varied for HGdA01, HGdA06 and GG06. The same applies to the optimal cloud factor threshold which is equal to 1 and hence is changed from 0.67 to 1.00.

To estimate whether parameter sensitivity is robust to different conditions, we performed the analysis separately for the following four conditions for Haut Glacier d’Arolla ablation season 2001: (1) day, (2) night, (3) clear-sky and (4) overcast. The sensitivity during daytime and night-time was determined by considering the melt in the hours 08:00–22:00 and 22:00–08:00, respectively. To distinguish between clear-sky and overcast days an algorithm developed by Carenzo and others (2009) was used. Since the conditions analysed are largely independent of the climatic setting, we do not expect large differences among sites, and we carried out the analysis only for HGdA01.

5. PARAMETER OPTIMALITY AT THE DISTRIBUTED SCALE AND MODEL UNCERTAINTY FOR HGdA01

To test that parameters are optimal at the distributed scale and quantify model uncertainty we use a Monte Carlo approach. The same method was employed by Anslow and others (2008) and Ragettli and Pellicciotti (2012). A total of 10^4 model realizations were obtained by randomly selecting model parameters from a uniform distribution out of a range vector with parameter changes from –5% to +5% around the optimal value (with a step size of 1%). By plotting the model performance associated with each model run we can evaluate whether the optimal parameter values at the point scale are also optimal at the distributed scale in the space searched by the Monte Carlo simulations. For every run, we plotted the total melt against the corresponding root-mean-square error, rmse, at the ablation stakes (Fig. 1).

The standard deviation, stdv, of the total melt for the 10^4 runs provides a measure of the uncertainty in model outcome that is associated with a particular amount of uncertainty in model parameters (Ragettli and Pellicciotti, 2012). Because stdv depends on the number of runs, we plotted it against the number of runs and verified that stdv stabilizes around a particular value within the 10^4 runs. We then took the mean of the stdv during the period after which no further important changes in stdv are evident.

To determine model uncertainty it is crucial to perform the 10^4 runs with parameters which are near their optimal value at the distributed scale. We therefore performed 10^3 preliminary runs with the optimal values calibrated at the point scale, validated them against the stake readings, identified a new optimal value and started the 10^4 realizations with the new parameter set.

In the assessment of the sensitivity of individual parameters (Section 4), the albedo parameters from the parameterization of Brock and others (2000) were found to be more sensitive than those of the US Army Corps of Engineers (1956) model. Therefore for the model uncertainty simulations, albedo was computed based on the parameterization of Brock and others (2000).

6. RESULTS

6.1. Sensitivity to individual parameters

Figure 4 shows the magnitude of calculated sensitivity for the most sensitive parameters (sensitivity >0.5 for basin section: glacier) for the different basin sections and the four daily melt statistics for HGdA01. Parameter sensitivity ranking is consistent across different basin sections and daily statistics, except for the minimum melt. The same results were obtained for all melt seasons and glaciers examined and are not reported here, for reasons of space. For HGdA01 the parameters to which the model is most sensitive to are the parameters related to the modelling of the shortwave-radiation flux, i.e. SRF (Eqn (1)), α_{ter} and α_{3us}, as well as cf_{1}. Minimum melt, by contrast, is controlled by the temperature factor, TF, of the melt equation (Eqn (1)). Since mean and maximum melt are more relevant than minimum melt for assessment of glacier melt, we concentrate on the model sensitivity with respect to these, and consider minimum melt no further here.

In Table 3, the top ten parameters to which the model is most sensitive are listed for the different melt seasons and glaciers. In contrast to glacier ice, the amount of snow on the slopes is limited during one ablation season and this affects parameter sensitivity. On the slopes, an increase in melt rate is reflected in parameter sensitivity only if additional snow is available for melt. Similarly, small reductions in melt rates might still be sufficient to melt all the snow if there is only a
limited amount, and therefore the model would be equally sensitive to the parameters. Therefore sensitivity on the non-glacierized slopes strongly depends on the amount of snow available, which in turn depends on the initial SWE and solid precipitation.

Uncertainty in the initial snow conditions or in the distribution of the ablation season precipitation might therefore affect the results. Hence we focus on the sensitivity in the basin as a whole and on the glacier where this influence is smaller. The parameter ranking for ‘daily mean’, ‘daily max’ and ‘daily stdv’ is, except for minor differences, the same for the basin, and the ranking for the basin is the same as for the glacier. Therefore only the ranking of ‘daily mean’ melt on the glacier is shown.

Results are consistent for the different sites (Table 3). Mean melt, together with its daily variation (stdv) and maximum melt, is controlled by the parameters influencing the shortwave-radiation-dependent term of the melt equation (Eqn (1)). There are some differences in the actual magnitude and ranking of the sensitivities, but the first eight parameters for all sites and seasons are the SRF, the albedo parameters, and the coefficients of the cloud factor parameterization (Table 3). In addition to these, important parameters are the LR for both Gornergletscher and Glaciar Juncal Norte and T_f for Glaciar Juncal Norte. For all the glaciers the empirical parameter, SRF, is among the most sensitive (Table 3). For Alpine glaciers, where summer precipitation is important, the model is sensitive to the albedo parameters, α_1br and α_1us, in contrast to JNG08/09, where precipitation is largely absent. The model is sensitive to cloud factor parameters across melt seasons and glaciers (Table 3).

On the non-glacierized slopes, where only snowmelt can occur, the albedo parameters, α_1br and α_1us, and the SRF are important, as well as LR, T_f and ph_t. The latter control the amount of snow available for melt, either by increasing the number of cells where melt occurs (LR and T_f) or by increasing the amount of solid precipitation (ph_t).

6.2. Sensitivity under different conditions

Figure 5 depicts the parameter sensitivity on HGdA01 for different conditions over one season (as well as for the entire period of record). Since the sensitivities for ‘daily mean’, ‘daily max’ and ‘daily stdv’ are very similar, only the sensitivity for ‘daily mean’ is shown.

During daytime, the sensitivity of the incoming shortwave-radiation-related parameters is slightly increased compared to the entire period of record. During night-time (22:00–08:00), I is absent for most of the hours and therefore the model is not very sensitive to I-related parameters. Model sensitivities under clear-sky conditions are basically the same as for the entire period of record (Fig. 5). Under overcast conditions, however, sensitivities are markedly distinct. The parameters α_1br, α_1us and SRF become less important (with a decrease in model sensitivity of ~15%), while the cloud factor parameters are more important (by ~25%).

6.3. Parameter optimality at the distributed scale and model uncertainty for HGdA01

Figure 6 shows total melt plotted against the corresponding rmse for every run, for HGdA01. The rmse is calculated by comparing modelled and measured melt at the ablation stakes and is used here as an indication of the best model.
performance. In Figure 6a the $10^3$ preliminary runs are depicted and Figure 6b shows the $10^4$ runs where the parameter set with the lowest rmse of the $10^3$ preliminary runs was used as the starting point.

The optimal set calibrated at the point scale is within one standard deviation of the optimum identified with the $10^4$ realizations (Fig. 6). Compared to the run with the lowest rmse (indicated by the triangle), total melt was overestimated by 4%. The parameter values calibrated at the point scale were compared to the parameter values of the new optimal run (i.e. the run with the lowest rmse in Fig. 6b). Considering all parameters, parameter values change on average by <3%, and considering only the most sensitive parameters (sensitivity >0.5 on the glacier) values change on average by only 2%. This is an indication that, albeit with some margin for improvement, the parameter set calibrated at the point scale is nearly optimal.

The development of the standard deviation (stdv) is shown in Figure 7, together with the mean standard deviation of the $10^4$ model realizations, for HGdA01. After ~6000 runs stdv starts to fluctuate around a mean value of $1.28 \times 10^3$ m$^3$ w.e., which corresponds to 5.6% of the mean total melt of the $10^4$ runs.

7. DISCUSSION
7.1. Sensitivity to individual parameters
One of the main findings of our work is that the model is most sensitive to parameters controlling the net shortwave-radiation flux. The much higher sensitivity of the shortwave-radiation-related parameters compared to temperature-dependent parameters depends partly on the nature of the model, in which the shortwave-radiation flux is explicitly included. However, it also indicates that the shortwave-radiation flux is the dominant source of melt energy for glaciers in the climatic settings considered, as found by numerous modelling studies (e.g. Greuell and Smeets, 2001; Willis and others, 2002; Pellicciotti and others, 2008). This is particularly so for glaciers in the dry Andes of Chile, where incoming shortwave radiation is very high during the melt season and clouds are basically absent in summer (Pellicciotti and others, 2008; Ragettli and Pellicciotti, 2012). It might be different for glaciers such as maritime ones, where temperature-dependent energy fluxes (e.g. the sensible heat fluxes) are higher (Giesen and others, 2008). These results seem to be robust, as sensitivity ranking is consistent across the different basin sections, different conditions and the daily statistics examined.

For Alpine glaciers with precipitation events in summer, the snow albedo parameters have a key influence upon the spatial and temporal evolution of melt rates, in agreement with, for example, Favier and others (2004) and Pellicciotti and others (2005). The same results were obtained by MacDougall and Flowers (2011) when analysing the sensitivity of an energy-balance model, even though the authors used a different albedo parameterization. Estimation of solid precipitation from point measurements is affected by uncertainties associated with both lack of knowledge about how best to extrapolate it in space and in its phase, as well as by undercatch errors at the gauges. For glaciers and seasons characterized by larger or more frequent summer precipitation it might be expected that the model is more sensitive to the parameters related to solid precipitation than revealed by the current analysis. However, this would not affect the ranking of the most sensitive parameters, as the model is already very sensitive to the albedo parameters (Table 3). Errors due to gauge undercatch might affect the sensitivity of the model to $T_p$ and $p_h$. Undercatch errors are difficult to evaluate in the absence of accurate measurements of wind at the gauge (Nespor and Svrcek, 1999; Zweifel and Svrcek, 2002) and these data were not available for Gomergletscher. Simpler constant correction factors would need calibration and would therefore compensate for any number of other errors, thus we did not use any in this study. Solid precipitation might therefore be underestimated and model sensitivity to the phase threshold and precipitation gradient might be higher. At our study sites, summer accumulation was relatively modest compared to total melt: summer accumulation is <5.4% of total ablation at the location of the AWSs at our three sites, and <7% for HGdA01 and HGdA06 when the wind-based correction for gauge undercatch of Zweifel and Svrcek (2002) is implemented (with total ablation calculated with an energy-balance model; Carenzo and others, 2009). The impact of the same correction on the summer mass balance at the distributed glacier scale is also minor, with differences in total ablation of 2.1% and 0.8% for HGdA01.
and HGdA06, respectively, if we include the correction of Carenzo (2012).

For glaciers such as GG06 and JNG08/09 with a wide altitudinal range, LR is among the parameters to which the model is most sensitive, in agreement with findings of Li and Williams (2008), Gardner and Sharp (2009), Petersen and Pellicciotti (2011) and Ragettli and Pellicciotti (2012). The high sensitivity of the model to the lapse rate and the increasing awareness that lapse rates that are uniform in space and constant in time are not appropriate for hydrological and glaciological studies (Braun and Hock, 2004; Li and Williams, 2008; Gardner and Sharp, 2009; Minder and others, 2010; Petersen and Pellicciotti, 2011) point to the need for further investigation of the spatial and temporal variability of air temperature, and the way this is represented in melt and mass-balance models.

7.2. Optimal parameters and model uncertainty

Using Monte Carlo techniques it is possible to generate model realizations by randomly sampling the space of the parameters in a given interval, to verify that the parameters identified as optimal with any of numerous available methods (from manual calibration to automatic techniques, from local to global optimization methods) are indeed optimal in a global sense (Iman and Helton, 1988). A key issue in this type of analysis is the number of model realizations that guarantees that the entire relevant space of plausible parameters is sampled (Finger and others, 2011). Even though the $11^{20}$ possible parameter set combinations (ten possible changes plus one initial value per parameter and 20 parameters) might seem too many compared to the $10^4$ model runs performed for HGdA01, Figure 6 shows that $10^4$ runs are enough to clearly develop the expected shape of a parabola and indicate that the shape is unlikely to change with additional runs. They also allow identification of the minimum in model error associated with the optimal parameter set. Although the identification of the optimal parameters depends on the dataset used (here stake measurements) and the optimization criterion (here mse) employed to compare the model simulation with the observations (Pellicciotti and others, 2012), our results indicate that the sensitive parameters calibrated at the point scale are also in the region of the optimal values at the distributed scale.

8. CONCLUSIONS

We have analysed the sensitivity of a distributed ETI melt model for a number of glaciers and seasons in the Swiss Alps and the central Andes of Chile, with the aim of identifying the parameters to which the model is most sensitive. By identifying the main parameters controlling the model behaviour, sensitivity analysis helps in deciding which parameters should be the focus of calibration (Tang and others, 2007). This is important in general, but especially in cases where only limited datasets are available for calibration, such as in the Andes of Chile and in the Himalaya (Pellicciotti and others, 2012). This type of investigation is particularly important to inform the design of field measurement campaigns (Tang and others, 2007), especially for models of snow and ice melt that often require devoted field campaigns at sites that are remote and difficult to access. In this light, such analysis is useful to identify the data that need to be collected to constrain model parameters that are relevant to the modelling. Also, despite the fact that the ETI model has been applied in various investigations in different regions of the world (Pellicciotti and others, 2005, 2012; Carenzo and others, 2009; Finger and others, 2011; Ragettli and Pellicciotti, 2012), this is the first study that analyses systematically, over several glaciers and seasons, its sensitivity to both model parameters and parameters controlling the extrapolation of the meteorological and surface variables used as input to the model.

Our main conclusions are as follows:

1. The parameters the model is most sensitive to are the shortwave-radiation factor, SRF, the air temperature lapse rate (especially for glaciers with a large elevation range), the albedo parameters (especially if precipitation is present), the cloud factor parameters and the temperature threshold (depending on the number of hours with temperatures close to $T_f$).

2. These results are consistent across different basin sections, different meteorological conditions over one season and the daily statistics examined, as well as across melt seasons and glaciers, with some differences, that are explained by the different climatic and topographic characteristics of the glaciers.

3. These results are useful for calibration strategies, as they indicate which parameters should be the focus of calibration and thus of the design of devoted monitoring campaigns. In particular, our findings suggest, in agreement with several recent studies, that the LR used for extrapolation of air temperature to the glacier or basin scale should be known with accuracy, as the model is very sensitive to it, especially for large basins. More measurements of air temperature variability over glaciers are therefore encouraged (Minder and others, 2010; Petersen and Pellicciotti, 2011).

4. Some of the parameters the model is most sensitive to are parameters of the equation used to calculate melt (e.g. SRF and $T_f$). Others, however, are parameters related to the generation of distributed fields of input meteorological and surface variables (e.g. air temperature, albedo and shortwave radiation). Our results are therefore likely to be valid for any distributed model that requires such input data (including energy-balance models), as confirmed by the findings of MacDougall and Flowers (2011), who also found a distributed energy balance was most sensitive to albedo parameters, and by those of Fitzgerald and others (2012), who found the same results for an enhanced version of a temperature-index model.

5. The parameter values calibrated at the point scale for HGdA01 seem to be in the region of the optimal values at the distributed scale. Monte Carlo techniques, such as the one employed in this work, are a useful tool to test that the identified model parameters are optimal, also because in most cases only limited datasets are available for parameter calibration.

6. Finally, a parameter uncertainty of 5% results in a model uncertainty of $1.28 \times 10^6$ m$^3$ w.e. for Haut Glacier d’Arolla, which corresponds to 5.6% of the mean total melt.
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