A GLOBAL OXYGEN ISOTOPE MODEL - SEMI-EMPIRICAL, ZONALLY AVERAGED

by

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ABSTRACT

A simple model, which is zonally averaged, for the transport of atmospheric water vapour is presented which uses as input the zonally averaged evaporation field and the mean meridional travel distance of tropospheric water vapour as functions of latitude. The model demonstrates that for polar regions each of the 10° latitude strips poleward of 25° is of equal importance as a moisture source. The model is used to predict zonal averages of $\delta^{18}O$ for the present day and 18 ka BP. Both annual average values and seasonal amplitudes are presented and compared to observations. Sea-ice cover is an important factor in determining both annual averages and seasonal amplitudes today and at 18 ka BP. An earlier model (Jouzel and others 1983) linking amplitudes today and at 18 ka BP is discussed, and sea-ice content in an Antarctic ice core to the relative humidity of the source region is based on a single-source atmospheric water-vapour cycle model and is re-evaluated using the present model.

INTRODUCTION

The $\delta^{18}O/\delta^{16}O$ ratio is given in terms of the unitless variable $\delta^{18}O$ (in $\%_{o}$) and referred to hereafter as $\delta$, and has been used extensively to study the movement of atmospheric water vapour. Oxygen isotope records from ice cores have been used previously to provide local temperature histories at polar sites (Dansgaard and others 1973, Barkov and others 1977, Budd and Morgan 1977, Paterson and others 1977, Lorius and others 1979). It is known that up to seven non-temperature effects can alter the $\delta$ content of precipitation at a given site (Dansgaard and others 1973, Fisher 1979), but the assumption has held that the local site temperature remains as a strong(est) signal in the $\delta$ time series. This assumption rests on a view that most water vapour arriving at polar sites is transported from the warm tropics (10 to 25°N or S), where evaporation rates and sea surface temperatures are high (Dansgaard and others 1973, Merlivat and Jouzel 1979). In the simplest form of the theory, isotopic fractionation en route follows the Rayleigh condensation process from a limited mass of vapour, and the $\delta^{18}O$ of the condensate at a site with temperature $T$ is

$$\delta = \left( F_v \left( \frac{\alpha_m}{\alpha_0} - 1 \right) \right) \left( 1 + \frac{\delta_0}{\delta_v} \right) + \delta_0 \tag{1}$$

where $F_v = y/v_0$ with $v_0$ and $y$ being the saturation mixing ratios of water vapour at the initial condensation temperature $T_0$ and site temperature $T$, and $\alpha_0$ and $\alpha_m$ are the fractionation coefficients (Majoube 1971) for $^{18}O$ at $T$, $T_0$ and $(T+T_0)/2$ respectively. $\delta_0$ is $\delta$ for the initial vapour mass at $T_0$ and is usually taken as $(1/\alpha_0-1)$ which is close to $-0.01$ (i.e. $-10^{\%/o}$). Other factors influence $\delta_0$ (Merlivat and Jouzel 1979), but for now we assume $\delta_0 = -10^{\%/o}$.

All the variables in Equation (1) are functions of either $T$ or $T_0$, thus $\delta$ itself is only dependent on the site temperature $T$, and the initial condensation temperature $T_0$ ($T_0 = +20^\circ$C for a sea surface temperature of $+25.4^\circ$C, with 80% humidity and adiabatic cooling of the moist air as it rises). The argument continues that $T_0$ in the tropics is very stable even under ice-age conditions, so that the $\delta$ value of precipitation is principally related to the site temperature $T$.

PROBLEMS WITH THE THEORY

Where difficulties with the single-source theory have been encountered, the concept of local water vapour mixing with the tropical has been introduced. For example, Koerner and Russell (1979) and Schriber (unpublished: 54-77) need to add about 20% local Baffin Bay water to the annual precipitation in the Coburg Island area (76°N, 79°30'W, just east of Devon Island) to account for a $\delta$ too positive for the air temperature. Some difficulties are harder to reconcile with the simple theory. There is evidence (England personal communication) that the regional air temperatures of north-east Ellesmere Island did not change sharply at the Wisconsin-Holocene transition at 10.5 ka BP. The $10.5^\circ$ step in $\delta$ in cores from the Agassiz Ice Cap (Fisher and others 1983) in this area could not have been due to a major sudden warming at the site. The air temperature of northern Ellesmere Island seems to have remained cold until at least 8 ka BP. Uplift curves, dated moraines and driftwood (England personal communication) suggest that ice retreat on northern Ellesmere Island began slowly in about 8 ka BP and speeded up around 6.2 ka BP.

Eriksson (1965) pointed out that the simple theory is physically unrealistic and that a global model of $\delta$ should depend directly on many meteorological variables along the whole water cycle (evaporation, precipitation, advective wind velocity, eddy diffusivity, temperature, relative humidity). He developed a set of equations containing all these variables, but data quality prevented him from solving them. Robin (1977) has examined and rejected the assumption that the region of major vapour input for polar sites is the warm tropics. He points out that little or no moisture injected equator-ward of 25° ever reached the polar regions. A number of authors have presented strong empirical evidence showing the importance of the distance from the closest moisture source in determining the $\delta$ value. For polar regions they have shown that the distance from open water is more influential than temperature in determining the $\delta$ value of the site (Kato and others 1978, Koerner 1979, Punning and others 1980, Bromwich and Weaver 1983).

THE MODEL

The following model uses zonal averages of evaporation, precipitation, temperature, net meridional vapour flux, and the position of the sea-ice front as input. All sources allowed by the net vapour-flux field contribute towards the precipitation at a given site. Since the model does not include orographic effects and is weighted towards ocean conditions, it generates results that apply to sea-level, island and coastal stations. The impetus of the work is to produce a model that includes the major variables in the water vapour cycle and
TABLE 1. ZONAL AVERAGES NORTHERN HEMISPHERE

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Area fraction 10° strip</th>
<th>Percent ocean</th>
<th>Ocean evaporation now (g cm⁻² s⁻¹)</th>
<th>Average ocean evaporation now*</th>
<th>Average air temperature now (°C)</th>
<th>Source weight now</th>
<th>Average temperature at 18 ka BP (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80-90</td>
<td>0.015</td>
<td>90</td>
<td>4.2</td>
<td>0</td>
<td>-23.6</td>
<td>5.7</td>
<td>-34.6</td>
</tr>
<tr>
<td>70-80</td>
<td>0.05</td>
<td>70</td>
<td>14.5</td>
<td>0</td>
<td>-15.9</td>
<td>51.0</td>
<td>-26.9</td>
</tr>
<tr>
<td>60-70</td>
<td>0.07</td>
<td>30</td>
<td>33.3</td>
<td>3</td>
<td>-7.2</td>
<td>70</td>
<td>-14.2</td>
</tr>
<tr>
<td>50-60</td>
<td>0.1</td>
<td>43</td>
<td>49.2</td>
<td>7</td>
<td>0.5</td>
<td>212</td>
<td>-6.0</td>
</tr>
<tr>
<td>40-50</td>
<td>0.12</td>
<td>48</td>
<td>87.8</td>
<td>15</td>
<td>7.5</td>
<td>506</td>
<td>1.5</td>
</tr>
<tr>
<td>30-40</td>
<td>0.14</td>
<td>57</td>
<td>153.5</td>
<td>20</td>
<td>14.0</td>
<td>1080</td>
<td>10.0</td>
</tr>
<tr>
<td>20-30</td>
<td>0.16</td>
<td>63</td>
<td>157</td>
<td>23</td>
<td>20.4</td>
<td>1583</td>
<td>18.9</td>
</tr>
<tr>
<td>10-20</td>
<td>0.17</td>
<td>74</td>
<td>152.8</td>
<td>25.5</td>
<td>25.1</td>
<td>1922</td>
<td>24.1</td>
</tr>
<tr>
<td>0-10</td>
<td>0.17</td>
<td>77</td>
<td>119.8</td>
<td>26.5</td>
<td>25.5</td>
<td>1568</td>
<td>24.5</td>
</tr>
</tbody>
</table>

*Dietrich and Kalle (1957)

generates reasonable δ values that are not just dependent on the site temperature.

Consider a target latitude zone width Δx_t (km) and distance x_t (km) from the equator. The moisture falling at x_t arrives from all the "upwind" zones. Assume for now that the source zones are closer to the equator than the target site, and let the amount of vapour over the target site x_t that originates from a source zone centered at distance x (from the equator) be y(x,x_t). Precipitation over the site removes some fraction k of all available vapour contributions. Then the total precipitation at x_t is

\[ P(x_t) = k \sum y(x,x_t). \]  

(2)

The δ of precipitation due to vapour from a source at x is δ(x,x_t) and the average δ(x_t) of all the precipitation at the target site x_t is

\[ δ(x_t) = \bar{δ} δ(x,x_t) \frac{y(x,x_t) k}{P(x_t)}. \]  

(3)

Now comes a key assumption. The moisture initially injected into the atmosphere at x, y_0(x) is taken to be proportional to the area of the source zone A(x), the fraction of ocean in the zone C(x), and the oceanic evaporation rate E(x). Thus y_0(x) = K.E.A.C., where K is a constant. The amount of vapour left at x_t from a given source at x is

\[ y(x,x_t) = K.E.A.C. f, \]  

(4)

where f = y/y_0, the depletion fraction. For brevity E.A.C. = F(x) and is called the source zone weight. Constants k and K have disappeared in the important weights in Equation (3)

\[ \bar{δ} F(x) \]  

\[ \frac{y(x,x_t) k}{P(x_t)} = \bar{δ} F(x). \]  

(5)

The \( F = E.A.C. \) weights for the northern hemisphere are obtained from annual zonal averages and appear in Table I for present-day conditions.

The ocean-weighted evaporation rate and the ocean fraction are used to calculate the F weights instead of the total zonal evaporation because the model is attempting to generate sea-level, island and coastal δ values. The assumption behind this is that the most precipitation at such sites originates from the oceans. This is reasonable because most of the northward net meridional vapour flux in the northern hemisphere is over oceans (Peixóto and Starr 1958: fig. 112). In practice the results are not very sensitive to whichever choice of E is used.

The depletion factor f is derived using the vapour survival time t(x), i.e. the average time that vapour at a given latitude stays aloft. During time t(x), vapour at x can travel some distance, whose meridional component is denoted \( \Delta(x) \), and can be thought of as the average meridional survival distance for water vapour at x. There are various estimates of t(x). We use that given by Junge (1963) based on t(x) = \( \frac{W(x)}{P(x)} \), where W is the amount of precipitable water and P is the precipitation rate. We define \( \Lambda(x) \) from the net annual zonally-averaged meridional vapour velocity \( \bar{V}_\phi \)

\[ \Lambda(x) = k \frac{V(x) \bar{V}_\phi(x)}{V_\phi} \]  

(6)

\( \bar{V}_\phi \) has been defined by

\[ \bar{V}_\phi(x) = \frac{sQ_\phi}{WS} \]

where \( \bar{Q}_\phi \) is the net annual zonally-averaged meridional vapour flux across a latitude line at distance x. W is the precipitable water vapour and S the peripheral length of the latitude circle at x. Included in \( \bar{Q}_\phi \) are both advective and turbulent, or diffusive, vapour transfer. Since \( \bar{Q}_\phi \) includes transport at all elevations the model will not produce elevation effects in the results. The model should be compared to sea-level observations only. Estimates of \( \bar{Q}_\phi \) and W come from Peixóto and Oort (1983) and are listed in Table II along with t(x) and \( \Lambda(x) \).

Figure 1 shows the mean annual meridional survival distance \( \Lambda(x) \) based on data for one year (Peixóto and Starr 1958). Positive values of \( \Lambda \) indicate northward vapour transport and negative values southward. This figure has two node points \( S_0 \) and \( N_0 \), where \( \Lambda(x) = 0 \). More recently published data by Peixóto and Oort (1983), based on data for ten years, show that \( \Lambda \) remains large and positive as far as 80°N. Their data also show the maximum of \( \Lambda \) in the northern hemisphere to be shifted...
TABLE II. ANNUAL MERIDIONAL SURVIVAL TIMES AND LENGTHS

<table>
<thead>
<tr>
<th>Latitude</th>
<th>Net meridional flux across a latitude band</th>
<th>Peripheral length</th>
<th>Precipitable water</th>
<th>Mean meridional vapour velocity</th>
<th>Mean residence time</th>
<th>Mean travel distance</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$sS\sigma$ (10^8 kg s^{-1})</td>
<td>S (10^7 m)</td>
<td>W (kg m^{-2})</td>
<td>$V_p$ (m s^{-1})</td>
<td>t (d)</td>
<td>$\lambda$ (km)</td>
</tr>
<tr>
<td>80</td>
<td>-0.11</td>
<td>0.69</td>
<td>4.8</td>
<td>-0.33</td>
<td>14</td>
<td>-399</td>
</tr>
<tr>
<td>70</td>
<td>0.54</td>
<td>1.37</td>
<td>6.1</td>
<td>0.65</td>
<td>10</td>
<td>562</td>
</tr>
<tr>
<td>60</td>
<td>2.36</td>
<td>2.0</td>
<td>9.6</td>
<td>1.23</td>
<td>8</td>
<td>850.4</td>
</tr>
<tr>
<td>50</td>
<td>5.32</td>
<td>2.57</td>
<td>13.3</td>
<td>1.56</td>
<td>6.3</td>
<td>850.5</td>
</tr>
<tr>
<td>45</td>
<td>5.80</td>
<td>2.82</td>
<td>15.3</td>
<td>1.34</td>
<td>6.4</td>
<td>742</td>
</tr>
<tr>
<td>40</td>
<td>5.46</td>
<td>3.06</td>
<td>17.7</td>
<td>1.01</td>
<td>7.5</td>
<td>653</td>
</tr>
<tr>
<td>30</td>
<td>2.91</td>
<td>3.46</td>
<td>24.0</td>
<td>0.35</td>
<td>10</td>
<td>302</td>
</tr>
<tr>
<td>20</td>
<td>-2.7</td>
<td>3.75</td>
<td>30.8</td>
<td>-0.23</td>
<td>11.5</td>
<td>-229</td>
</tr>
<tr>
<td>10</td>
<td>-6.16</td>
<td>3.93</td>
<td>38.3</td>
<td>-0.41</td>
<td>9.5</td>
<td>-336</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>3.99</td>
<td>43.9</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Fig.1. The zonally-averaged annual meridional survival distance of water vapour. $S_0$ and $N_0$ are the node points where in theory the moisture goes up and all comes down on the same spot.

south, and the $S_0$ point to remain as it is. Under the formalization of this model precipitation at $S_0$ all originates locally. In reality, the nodal point moves and precipitation at $S_0$ comes from sources north and south of $S_0$. The following approximation is used to estimate $\lambda(x)$ north of $S_0$ in Figure 1:

$$\lambda(x) = \lambda^* \left[ 1 - \left( \frac{x - x^*}{B} \right)^2 \right]$$

where $x^*$ is the symmetry point (-5820 km), $\lambda^*$ is the maximum $\lambda$ (= 1450 km), and $B$ is half the width between the intercepts of the parabola (= 3600 km). The $\lambda^*$ value from Figure 1 is seen to be 900 km, but the larger value above is used (the figure of 900 km is a total zonal average). Over the oceans the northward vapour flux is larger and thus the $\lambda^*$ appropriate to the ocean-weighted model is larger, besides by experiment the constants listed for Equation (7) give the best results. The constants of Equation (7) result in positive values of $\lambda$ between 21 and 85°N. If one could follow an amount of water vapour $y_0(x)$ originating from a zonal band centered at $x > S_0$, it would move northward and be depleted according to

$$\frac{dy}{dx} = -y/\lambda(x).$$

If $\lambda$ were a constant one would get the simple e-folding depletion relation $y/y_0 = e^{-x/\lambda}$. With $\lambda(x)$ given by Equation (7) one gets the depletion fraction at target $x_t$ of vapour originating at $x$:

$$f(x,x_t) = \frac{y(y_0)}{y_0} = \left[ 1 - \left( \frac{x - x^*}{B} \right)^2 \right] \frac{B/\lambda}{B/\lambda^*}$$

$$f(x,x_t) = \frac{1 - (x^* - x)/B}{1 + (x^* - x)/B}$$

The relative depletions $f = y/y_0$ go into Equations (5), (3), and (1) so that $\delta$ at site $x_t$ can be calculated. The values of $\alpha$ of Equation (1) are changed here to apparent or effective fractionation coefficients $\alpha_0$ (Eriksson 1965, Merlivat and Jouzel 1979) by

$$\alpha_0 = \alpha / \left\{ 1 + \alpha - 1(y_e/y) \right\}$$

Here $\alpha$ is "corrected" to take account of the unprecipitated liquid drop content $y_e$ in the air mass. $y_e/y$ is typically 0.2 (Eriksson 1965) and is a constant here. Junge (1977) points out, however, that $y_e/y$ can vary from 0.1 in the tropics to
over 0.5 in polar areas. The relative zonal-average precipitation rates can be calculated from Equations (9), (2) and (4).

APPLICATION OF THE MODEL

Figure 2 displays the $f_3'$ and $S(x,x_t)$ functions for a target zone centered at $75^\circ$N. The $f_3'$ curve shows the relative contributions of the source zones to precipitation at $75^\circ$N, and demonstrates that all the source zones between 30 and $75^\circ$N supply significant amounts of vapour to $75^\circ$N. In particular, the "local water" zone between 65 and $75^\circ$N contributes about 25% of the total precipitation. This fraction is in agreement with the conclusions of Koerner and Russell (1979) and Schriber (unpublished) about the origins of precipitation at sea level for northern Baffin Bay.

Fig.2. The dashed curve is the $S$ of modern precipitation contributions arriving at $75^\circ$N from various $10^\circ$ latitude strips and the solid curve is the $f_3'$ product.

Fig.3(a). Predicted $\delta$ vs. temperature for coastal and island sites from Yurtsever (1975): solid circle, Dansgaard and others (1973); open circle, Koerner and Russell (1979); solid triangle, and M Jeffries (personal communication); solid square. The solid lines are the predictions using as temperatures the zonal averages from Sellers (1967). The dashed line is the ice cap temperature relationship of Dansgaard and others (1975). The upper and lower solid lines assume smooth and rough source oceans, respectively.

Fig.3(b). Predicted annual average $\delta$ for present conditions $6^\circ$K and hypothetical conditions $18^\circ$ka BP, both as functions of latitude. Inset shows predicted ($\delta_{6K} - \delta_{18K}$) as function of latitude. The shaded portion of the difference is that due to the proposed shift in the annual mean position of the sea-ice front.

Fig.3(c). Precipitation (normalized) zonal averages. The dashed line represents measured values; the solid line is the model prediction for present conditions.

Figure 3(a) shows that values of $\delta$ estimated from the model (solid line) run through the measured points ($\delta$, temperature) from island and coastal stations (solid circle: Yurtsever 1975; open circle: Dansgaard and others 1973, coastal sites), and some coastal sites in the Canadian Arctic Islands. Comparison to marine values of $\delta$ is appropriate because of the model parameters used and the dominant meridional vapour flux being mainly marine. The points from Yurtsever's plot that are described as "suffering" from the amount effect (Dansgaard 1964) are not
moisture from sources between 75° to about 83°N, this region being mostly land.

Figure 4 summarizes the measured seasonal amplitude $A = \left( \text{summer} - \text{winter} \right)/2$ for Greenland and Canadian Arctic ice cap stations, coastal stations, and some Antarctic sites. The dashed lines are linear regressions through the northern ice-cap and coastal stations. The solid line is the model prediction. The model line follows the coastal station line (lower dashed) until about 63°N, and then rises to meet the ice-cap line near 75°N where $A$ reaches a maximum for both model and ice cap. The contribution of the sea-ice cycles to $A$ can be seen immediately by comparing coastal stations at either limit of the sea-ice cycle. At Gronnedal (61°N), which feels little or no sea-ice effect, $A = 3.3°$ and at 75°N, which gets the full effect, $A = 5.8°$. The maximum contribution to $A$ of a 15° latitude march in sea ice is about 3.1°, about half of the total $A$ at 75°N. With the rough seasonal parameters used here the model predicts a 4.8° contribution to $A$ from the sea-ice cycle and 4.7° from the other seasonal variables. The rise of $A$ in the model above the measured coastal line north of 65°N is possibly because the seasonal values of $E$ used are not realistic north of 60°N; also one cannot hope to reproduce much more than general tendencies from a zonal model even if it is "tuned" to a particular area. The "odd" small measured values of $A$ for the coastal stations south of 80° could be related to the presence of the $N_2$ mode at 77°N. Sites north of 80° receive much of their moisture from the Arctic Ocean.

Fig. 4. The latitude dependence of the amplitude of the seasonal cycle $A = \left( \text{summer} - \text{winter} \right)/2$. The solid line is the model prediction. The following equations represent measurements made at high-elevation ice cap sites from Greenland and the Canadian Arctic: solid circle, coastal sites from Greenland; open circle, Antarctic sites; open square. The (lower) dashed line is a least-squares linear fit to the coastal sites and the (upper) dot-dashed line is for the ice cap sites. Adapted from Fisher and others (1985).

MODEL APPLIED TO HYPOTHETICAL ICE-AGE CONDITIONS

In order to try out the model on some hypothetical conditions 18 ka ago, we assume that the annual average $\lambda (x)$ is 18° warmer than today's sea-ice-free temperature for the Atlantic Ocean (CLIMAP Project Members 1976). In the high Arctic the 18 ka BP temperature is obtained from the climatic $\delta$ shift of the Devon Island ice core (Paterson and others 1977). This 18° shift suggests a temperature 11°C cooler than today. Although this use of $\delta$ to estimate the 18 ka BP temperature is circular, we note that the 11°C fits in as a reasonable northward extrapolation of the CLIMAP ocean temperature changes. Also, the values of $\delta$ in the model at high Arctic target sites are not markedly sensitive to the temperature at the target site. These 18 ka BP temperatures are listed in Table I. In addition, we assume an 18 ka BP sea-ice cycle having the January ice front at 55°N and July at 65°N. The annual average position is about 10° farther South than today. The calculated mean annual values of $\delta$ for Syowa in Figure 3(b) plotted against latitude, and the inset shows the ($\delta_{689} - \delta_{618}$) difference. The $\delta$ differences are about the right amount.
Fig. 5(a). $\delta^{18}O$ vs. $\delta$(D). The model produces the solid line. The dashed line represents Dansgaard's (1964) empirical line, small circles are Dansgaard's points, solid squares are Byrd station ice-core averages (Epstein and others 1970), large open circle is Dome C ice core, adapted from Jouzel and others (1982), and heavy closed circle is Horlick Mountains (Dansgaard 1964).

Fig. 5(b). The southern-hemisphere predicted deuterium excess $\delta$(D) - $\delta^{18}O$ vs latitude for smooth ocean curves, upper are for a relative humidity of 80% and the lower of 90%. The measured southern hemisphere values are symbolized as in Figure 5(a).

The southern-hemisphere predicted deuterium excess $\delta$(D) - $\delta^{18}O$ vs latitude for smooth ocean curves, upper are for a relative humidity of 80% and the lower of 90%. The measured southern hemisphere values are symbolized as in Figure 5(a).

Table III. Ice core $\delta$ differences ($\delta_{\text{D}} - \delta_{\text{O}}$)

<table>
<thead>
<tr>
<th>Site</th>
<th>Latitude north</th>
<th>Present elevation (m a.s.l.)</th>
<th>($\delta_{\text{D}} - \delta_{\text{O}}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agassiz Ice Cap</td>
<td>81</td>
<td>1670</td>
<td>8</td>
</tr>
<tr>
<td>Camp Century, Greenland</td>
<td>77</td>
<td>1890</td>
<td>11.5</td>
</tr>
<tr>
<td>Devon Island ice cap</td>
<td>75</td>
<td>1830</td>
<td>7</td>
</tr>
<tr>
<td>Barnes Ice Cap</td>
<td>70</td>
<td>-900</td>
<td>19-15**</td>
</tr>
<tr>
<td>Dye 3, Greenland</td>
<td>65</td>
<td>2460</td>
<td>7***</td>
</tr>
</tbody>
</table>

* If, as we believe (Fisher and others 1983), a wind scrou correction should be applied to all $\delta$ values older than 3500 BP, the difference is 10.5%/o.
** Contains a large flow-line effect (see text); bracketed values are flow-corrected.
*** Dansgaard and others (1982).

where, as before, $y$ is the mixing ratio of water vapour, $y_a$ is the mixing ratio of liquid droplets in the clouds, and $\alpha$ is the fractionation coefficient for $\delta^{18}$O or D.

Their expression for the initial vapour $\delta$ is

$$\delta_0 = \frac{1}{\alpha_c} \left[ \frac{1-k}{1-k h} \right] - 1$$

where $\alpha_c$ is the isotopic fractionation coefficient ($\delta^{18}$O or D) at the sea-surface temperature and $h$ is the relative humidity in the air mass over the ocean. $k$ is the variable related to kinetic (non-equilibrium) fractionation effects in the air-ocean boundary layer. For $\delta^{18}$O

$$k_{18^O} = 7^\circ/o_o$$

for smooth oceans

and

$$k_{18^O} = 4^\circ/o_o$$

for rough oceans

(Merlivat and Jouzel 1979: fig.2). In addition, they show that for deuterium D

$$k_{D}k_{18^O} = 0.88$$

They define a rough regime as having wind speeds $>7$ m/s at 10 m above the surface. Although they quote Eriksson and Bolin (1965) as stating that 95% of the world's oceans are smooth, they calculate an ocean-weighted value for $k_{18^O}$ of 4.8%/o. The present model was run with two $k_{18^O}$ extremes of 7%/o and 4%/o.

The independent variable in the integration is $x$, the distance from the equator, and Equation (11) is used in the form

$$\frac{d \delta}{dx} = (1 + \delta) \left[ \frac{\alpha (\alpha - 1) dy + y \alpha}{\alpha (y + \alpha y_a)} \right]$$

with

$$\frac{d \alpha}{dx} = \frac{\alpha dT}{dT dx}$$

(11)
T being the average zonal temperature and \( y_0/y \) as before is kept at a constant 0.2. \( dy/ydx \) comes from\( (7) \) and \( (8) \). From Jouzel's (1962) (squares), and Dome C (Loriüs and others 1981) (ellipses) least squares line (Fig.5(a)) \( (D) = 8.1 \) \( (80^o \text{O}) + 11 \). The model points all fall very nearly on their solid least squares line (Fig.5(a)) \( (D) = 8.37 \) \( (80^o \text{O}) + 11.9 \). The deuterium excess is defined by \( d = (D) - 8 \) \( (80^o \text{O}) \).

Jouzel and others (1982) examined the important variables influencing \( d \) in the ice core recovered from Dome C, Antarctica. There is a decrease of 4.6% in \( d \) from Holocene to ice age. They concluded that (i) the \( d \) excess is conserved from source to target, (ii) \( d \) is mainly determined by the relative humidity \( h \) of the air over the source ocean, and (iii) the \( d \) shift of 4.5% in \( d \) from present to ice age is thus due to a change in relative humidity from 80 to 90%.

Using the modern atmospheric and oceanic data for the southern hemisphere and an annual average sea-ice front at 65\(^\circ\)S (Ackley, 1981), the present multi-source zonal model calculates the deuterium excess shown in Figure 5(b) for 80 to 90% relative humidity and for smooth and rough source oceans. Since both \( h \) and \( k \) are probably latitude variables the actual \( d \) values are likely to be some average of the curves. The sparse "measured" values of \( d \) for the southern hemisphere are plotted. Since the model model calculates and matches the sea-level data, the model values of \( d \) are not really valid for latitudes south of 70\(^\circ\)S. It seems from Figure 5(b) that \( d \) presently has a latitude dependence and, furthermore, at any given site \( d \) can be shifted as much as 8% by altering \( k \) and \( h \) within reasonable limits.

Speculative runs of the model for the southern hemisphere were made assuming ice-age (18 ka BP) air and sea temperatures altered by amounts suggested by CLIMAP Project Members (1976: fig.3) and Robin and Johnsen (1983: chapter 6.3) and taking an average ice-age sea front at 55\(^\circ\)S. The calculated \( d \) values suggest that half of the 4.5% \( d \) change at Dome C could be related to the latitude dependence of \( d \). Because of other evidence presented by Jouzel and others (1982) (i.e. higher ice-age salt concentrations and atmospheric model predictions of enhanced storminess), it seems that \( k \) and \( h \) air latitudinal and longitudinal spreads are the \( d \) shift. Assigning all the \( d \) shift to an \( h \) change seems to be arbitrary. Of course, a zonal model can only suggest what variables are important in affecting \( d \) at any given site. For example, Figure 5(b) shows a change in \( d \) of the Byrd station core of about 8% at 60\(^\circ\)S. The zonal model cannot explain such differences from one longitude to another. Finally, the calculated (65\(^\circ\)A-68\(^\circ\)A) differences on the coast of Antarctica are about 4% at 60\(^\circ\)S, or about half those calculated for northern ice-cap sites. This is in agreement with the ice-core data.

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