An alternate statistical interpretation of the strength of snow: comments on the paper by H. Gubler

Gubler (1978[a], [b]) presents very interesting work which may lead to a greater understanding of the physical processes involved in snow failure. However, he raises a point in the second paper which I do not believe to be valid. To quote Gubler “From the strong dependence of the strength distribution on sample size, it follows that the link definition of Sommerfeld is very critical”. In fact, extreme-value statistics show that the large-volume strengths of materials do not depend much on sample size, but are critically dependent on the distribution of the weakest strengths in the material. Gubler's calculations show this very clearly, as did Epstein's (1948) analytical work. The way different distributions tail off at the low end critically determine the predicted large-volume strengths. If the samples are large enough, the volume dependence of the strength is low.

Actually, the most accurate way to determine the large-volume strength of a material is to measure the strengths of large volumes, not as Gubler seems to imply, very small volumes. In large-volume measurements, all questions of detailed material structure, like Gubler's "link definitions" are necessarily ignored. Large volumes of snow are very difficult to handle and it would be very convenient if the large-volume strengths could be derived from measurements on smaller volumes. As I have shown (Sommerfeld, 1974) practical test-sample sizes probably do not measure the lowest strengths accurately. These inaccuracies result not from theoretical definitions but from the practical consideration of sample-wall interference with material flaws.

Concerning shear strengths, Gubler asks “But what happens if the measurements are performed with a different shear-frame size?” One answer, of course, is that Daniels' statistics predict that the larger the frame size the lower the mean strength and the lower the standard deviation of the measurements. If Daniels' statistics actually apply to snow, the same large-volume failure stress would be predicted no matter what the test-sample size. This is clearly seen by considering Daniels' example of a bundle of a large number of threads. The bundle will fail at some stress. This stress is predicted, according to Daniels, by sampling the strengths of the threads. It does not matter if we consider the strength distribution of single threads or, for example, pairs of threads. If the theory is correct, the predicted failure stress for each analysis would be correct and both equal to the actual failure stress. This is true of the prediction up to the trivial case of the sample consisting of one test on the whole bundle.

A properly designed test using different size samples would be one way to test the applicability of Daniels' statistics to snow. Perla (1977) presents one such experiment comparing 25 tests each with 0.01 m² and 0.25 m² frames. He found a decrease in mean strength and standard deviation as predicted by Daniels, but did not determine the Daniels strength for each case.

Both Daniels' and Gubler's use of integral expressions implies they are dealing with a continuous medium and not a body made of discrete elements. With a large number of "threads" or "fundamental units", the distinction is not important, but then, neither is the exact character of the elements so long as they are described accurately enough by the chosen distribution.
For these reasons, I strongly disagree with Gubler’s implication that the test-sample size is theoretically critical to the determination of large-volume strengths. I do agree that for extreme-value predictions of large-volume strengths, the exact distribution of the lowest strengths is critical. If Gubler’s analysis could lead to the determination of which of all possible distributions is the “natural” or most accurate one, more confidence could be attributed to the predictions. One result from Gubler’s definitions of fundamental units is that it eliminates the normal distribution from consideration. If “(b) Each fundamental unit acts as a force-conducting element in the snow”, then, by definition, there can be no fundamental units with zero strength since strength is needed to conduct force. The normal distribution with its finite probability of zero strength is thus eliminated. The field is not narrowed very much since it is still open to the log-normal and a wide variety of truncated distributions, but it does appear to me that a step has been made.

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REFERENCES


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An alternate statistical interpretation of the strength of snow: reply to comments by R. A. Sommerfeld

I agree with the first comment of R. A. Sommerfeld that it would be very convenient if the large-volume strength could be derived from measurements on smaller volumes. I only showed (Gubler 1978[a], [b]) that the extrapolation to larger volumes depends strongly on the link-strength distribution chosen. Sommerfeld remarks that technical reasons impede a determination of the exact distribution type for the strength of the fundamental units or test samples from field measurements. But if future experiments allow the determination of the strength distribution of the microscopic links defined by Gubler (1978[b]), an exact extrapolation from measured smaller-volume strength to large volumes would be possible. (If the snow under investigation is homogeneous in a macroscopic sense.) Concerning shear strength and Daniels’ statistics, Sommerfeld seems to imply that his test samples are not conclusively identical with the fundamental units. So each test sample may consist of an unknown number of fundamental units. If the link number per test sample is high enough, Daniels’ theory predicts a constant expectation for its strength independent of the number of links per sample. If Sommerfeld’s test samples consist only of several links, he has to develop a method which enables him to determine the original link-strength distribution. Daniels’ suppositions clearly require a logical definition for the links. The theory implies the existence of only two states of a link: completely broken or surviving. But Sommerfeld’s test volumes may break in part during natural stress increase showing that they cannot be considered as the fundamental links. For these reasons, it seems to me that one has to know the strength distribution of the logical links in order for it to be permissible to apply Daniels’ statistics. There still remains a second problem: the stress-rates applied to the test samples are at least three orders of magnitude higher than the natural stress-rates. This fact together with the well-known high stress-rate dependence of strength of snow indicates that it is not possible to determine ductile shear strength using the sampling