MECHANICAL ENERGY CONSIDERATIONS IN SEA-ICE DYNAMICS

By M. D. Coon and R. S. Pritchard

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ABSTRACT. An equation describing the balance of mechanical energy is developed by forming the inner product of the ice velocity and the momentum-balance equation. This relationship shows how air stress acting on a moving ice cover transmits energy into the ocean. The mechanical energy relationship is valid for understanding ice behavior using mathematical models and is independent of the choice of constitutive law. By an example using a plastic ice model, the effect of the ice cover is identified. The energy stored and dissipated by the ice and the energy transmitted into the ocean are expressed as fractions of the energy available when ice is present but strength is negligible (free drift). The example represents an idealized, but realistic, description of winter conditions off the north slope of Alaska in the Beaufort Sea. Under the simualated conditions, the ice stress prevents up to 73% of the energy available in the atmosphere from reaching the ocean. About 34% of the energy available is dissipated by the ice through ridging, all of which occurs near the shore. The mechanical energy equation is expected to provide a powerful tool for understanding and simulating sea-ice behavior.

INTRODUCTION

Motion of the ice cover on polar oceans is affected by winds through air stress, by water stress and ocean currents, by Coriolis force, by internal ice-stress divergence, by sea-surface tilt, and by inertia. The balance of momentum in the plane of the ocean surface is satisfied by considering these forces. In the present paper, an inner product of the ice velocity and the momentum-balance equation is formed. This procedure allows the driving forces and ice response to be interpreted in terms of work done on the ice by the various forces. The resulting equation describing balance of mechanical energy is useful because it helps understand the processes by which the ice cover affects the transfer of energy from the atmosphere into the ocean.

RESUMÉ. Considérations sur l'énergie mécanique dans la dynamique de la glace de mer. On établit une équation décrivant le bilan de l'énergie mécanique en formant le produit interne de la vitesse de la glace et l'équation de la conservation des quantités de mouvement. Cette relation montre comment la force de l'air agissant sur une couverture de glace mobile transmet son énergie à l'océan. La relation d'énergie mécanique est utile pour comprendre le comportement de la glace à partir de modèles mathématiques et est indépendante du choix de la loi d'écoulement. Sur un exemple utilisant un modèle plastique pour la glace on retrouve l'effet de la banquise. L'énergie stockée et dissipée par la glace et l'énergie transmise à l'océan s'expriment comme des fractions de l'énergie disponible lorsque la glace existe, mais sa résistance est négligeable (poussée libre). L'exemple figure une description simplifiée mais réelle des conditions hivernales au large du versant Nord de l'Alaska dans la mer de Beaufort. Dans les conditions reconstituées la glace préleve jusqu'à 73% de l'énergie disponible pour atteindre l'océan. Environ 34% de l'énergie disponible est dissipée par la glace en créant des ondulations qui se produisent toutes près du rivage. On attend de l'équation d'énergie mécanique qu'elle fournisse un outil puissant pour comprendre et simuler le comportement de la glace de mer.


INTRODUCTION

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The air stress acting on the moving ice cover transmits energy into the ice-ocean system. A portion of this energy is dissipated by deformation of the ice and the remainder is transmitted into the ocean. At times the energy is transmitted into the ocean locally, while at other times the energy is transported long distances horizontally before its effects are observed. The equation for the balance of mechanical energy describes the relationship between these terms. This relationship is as fundamental as the momentum-balance equation since it depends only on the momentum-balance principle. Although the models that describe sea-ice dynamics may differ in their constitutive laws, they all consider essentially the same set of forces in momentum balance. Therefore, the equation of mechanical energy balance is applicable for any model.

An equation expressing the balance of mechanical energy has been derived for a variety of areas of scientific and engineering study. The concept has served well in such diverse areas as particle mechanics (Synge, 1960), fluid mechanics (Bird and others, 1960), upper ocean dynamics (Bryan, 1969; Phillips, 1977; Holland, 1978), and continuum mechanics (Truesdell and Toupin, 1960). The results obtained in this paper represent an application of the general concept to a specific problem—understanding the large-scale behavior of the ice cover.

An example problem is analyzed to demonstrate one way in which the mechanical energy-balance equation may be used. For this example, an elastic-plastic constitutive law is assumed, but the energy concept is valid for any constitutive law. The problem is intended to represent idealized, but realistic, winter conditions of ice behavior off the north slope of Alaska in the Beaufort Sea.

MECHANICAL ENERGY BALANCE

A mechanical energy balance for the ice cover is found by forming the inner product of the momentum-balance equation and the ice velocity. This approach was chosen because the primary forces that affect ice dynamics are known. A more comprehensive approach could be taken by stating all the terms that can affect energy conservation, including thermodynamic terms. The primary efforts of this study, however, are aimed at interpreting the various terms that affect the mechanical energy balance for short time intervals, say 1 to 5 d, when thermal growth and ablation are unimportant.

In the plane of motion of the sea ice, the momentum balance is expressed as

\[ m \dot{v} = -mfk \times v + \nabla \cdot \sigma + \tau_a + \tau_w - mg \nabla H \]  
(1)

(see Rothrock, 1975[b]), where \( m \) is the mass per unit area, \( \dot{v} \) is the material time rate of change of \( v \), the ice velocity, \( \tau_a \) and \( \tau_w \) represent the tractions exerted by the atmosphere and ocean on the upper and lower surfaces of ice, \( f = 2 \Omega \sin \phi \) is the Coriolis parameter at latitude \( \phi \), \( \Omega \) being the Earth's rotation rate, \( k \) is the unit vector in the vertical direction, \( \sigma \) is the Cauchy stress resultant in excess of hydrostatic equilibrium (two-dimensional), \( g \) is the acceleration due to gravity and \( H \) is the dynamic sea-surface height.

Upon forming the inner product of the ice velocity and the momentum-balance equation, a local form of the balance of mechanical energy is obtained:

\[ \kappa = p_a - p_w - p_l \]  
(2)

where the kinetic energy (areal) density of the ice cover,

\[ \kappa = \frac{1}{2} m \tilde{v} v, \]

is driven by the atmospheric power-density input,

\[ p_a = \tilde{v} \tau_a. \]  
(3)

In all expressions \( \tilde{v} \) is the transpose of \( v \). The ocean in turn is driven by power transmitted through drag and tilt in the form

\[ p_w = -\tilde{v} \tau_w + mg \tilde{v} \nabla H. \]  
(4)
The remaining term represents the influence of internal ice stress on the transfer of energy from the atmosphere to the ocean:

\[ \dot{p}_i = -\nabla \cdot (\nabla \sigma). \]  

(5)

The signs have been chosen so that the terms are positive when the atmosphere is working on the ice in the amount \( p_a \), the ice is being worked on at the rate \( \dot{p}_i \), and the ocean is receiving at the rate \( p_w \). It is perhaps worthwhile to note that at certain times the deep ocean currents provide the primary driving force and may move the ice in opposition to the wind. This case is accounted for properly by the mechanical energy-balance relationship where individual power terms become negative.

Although the title of this paper and the form of Equation (2) imply that we shall consider the energy of the atmosphere–ice–ocean system, the behavior of the ice dictates that it is actually the power that must be considered. This arises because the kinetic energy of the ice is an unimportant term on time scales of \( 1 \) d and longer. Accelerations are small enough for inertia and changes in kinetic energy to be negligible (see Thorndike and Colony (in press), for the power spectrum of the ice velocity). Therefore, the system remains in a state of quasi-steady equilibrium in which the rate of work done by the atmosphere \( p_a \) is balanced by the effect of the ice stress \( \dot{p}_i \) and the rate of work done on the ocean \( p_w \).

The effect of the ice stress is separated into two parts:

\[ \dot{p}_i = -\nabla \cdot (\nabla \sigma) + \dot{p}_d, \]  

(6)

where \( \dot{p}_d = \nabla \cdot (\nabla \sigma) \) is the divergence of the stress flux and \( \dot{p}_d = \text{tr} \sigma D \) is the trace of the product of stress and stretching. The stress flux represents the local imbalance of the energy transfer from the atmosphere to the ice and ocean, and it provides a description of the horizontal transport of energy by the ice from one location to another. The term \( \dot{p}_d \) represents the energy that is dissipated or stored in the ice cover. The example presented in a later section of this paper shows that power input over a wide area of the ice cover is dissipated in a smaller region near the shore.

Since kinetic energy is negligible for ice dynamics on space scales on the order of tens of kilometers and time scales on the order of \( 1 \) d, the local mechanical energy balance can be approximated as

\[ p_a - p_w + \dot{p}_i - \dot{p}_d = 0. \]  

(7)

Over a finite region of the ice cover it is desirable to learn the relative amounts of energy transmitted from the atmosphere, dissipated by the ice, transported by the stress flux, and received by the ocean. Averaging over the typical region \( R \) shown in Figure 1, we find

\[ P_a - P_w + P_i - P_d = 0, \]  

(8)

where each term \( P \) represents an average of \( p \) over \( R \). That is

\[ P = \frac{1}{A} \int_R p \, da. \]

(9)

Fig. 1. Two-dimensional region of ice \( R \) bounded by the curve \( \Sigma \). Figure shows at one point the air stress \( \tau_a \) on the top surface, the velocity \( \mathbf{v} \), and the water stress \( \tau_w \) acting on the bottom surface. The traction \( \mathbf{t} = \sigma n \) acts on the boundary.
The average stress flux is transformed by the Green–Gauss theorem to the form

$$P_t = \frac{1}{A} \oint_{\partial \mathcal{R}} (\mathbf{\tilde{v}} \mathbf{\sigma}) \cdot \mathbf{n} \, dl,$$

and therefore is seen to be the rate at which work is done on the boundary of the region $\mathcal{R}$ by the traction $t = \mathbf{\sigma} \cdot \mathbf{n}$. Equation (8) is seen to be a simple balance between the rates of work done on the ice at its top surface $P_t$ and boundary $P_f$, and the power dissipated by the ice $P_d$ and the rate of work on the ocean $P_w$.

The influence of the internal stress in the ice is demonstrated simply by examining the mechanical energy balance that would occur in free drift. This analysis is accomplished by neglecting divergence of ice stress in Equation (1). Thus, when inertia is negligible,

$$-m \mathbf{f} \times \mathbf{v}_{fd} + \mathbf{\tau}_a + \mathbf{\tau}_{wfd} - mg \nabla H = 0,$$

where $\mathbf{v}_{fd}$ is the free-drift ice velocity. The water drag depends also on $\mathbf{v}_{fd}$. Defining $P_{a\text{fd}}$ and $P_{w\text{fd}}$ by analogy with Equations (3) and (4) respectively, we find that

$$P_{a\text{fd}} - P_{w\text{fd}} = 0. \tag{12}$$

The average over region $\mathcal{R}$ follows directly from Equation (9) as

$$P_{a\text{fd}} - P_{w\text{fd}} = 0. \tag{13}$$

To illustrate how Equations (8) and (13) can be used to study sea-ice problems, attention is turned to Figure 2. The curve describing $P_{a\text{fd}}$ shows that for free drift the amount of energy removed from the atmosphere is transmitted to the ocean. The balance of terms in Equation (8) is shown in Figure 2 by presenting $P_a$ and $P_w$. Therefore, the shaded area represents $P_1 = P_d - P_t$. The effect of the ice stress on the energy transmitted from the atmosphere to the ocean is shown by the difference in $P_{w\text{fd}}$ and $P_w$. If an ice cover with no strength were present, then $P_{w\text{fd}}$ would be transmitted to the ocean; however, with the effect of the ice cover considered, only $P_w$ is transmitted. The difference between $P_{w\text{fd}}$ and $P_w$ shows how much less energy is taken from the atmosphere because of the strength of the ice cover. This will be discussed more in the example which follows. In Figure 2, the times when the ice cover is in free drift ($t_1 \leq t \leq t_2$) are shown by all the curves having the same value. The case when no energy is transmitted to the ocean ($t_1 \leq t \leq t_4$) is also shown. At these times the ice cover is strong enough to stop the ice velocity completely even though there is a significant applied air stress (shown by $P_{a\text{fd}} > 0$). Although $P_d - P_t$ is the effect of the ice stress, it is seen that $P_{w\text{fd}} - P_w$ also provides a measure, and both independently provide valuable information in understanding sea-ice behavior. The latter of these expressions, $P_{w\text{fd}} - P_w$, is of value for analyzing drift data to determine the importance of ice stress on the response. This difference depends only on a knowledge of the winds and resulting ice drift-rates and may therefore be determined without assuming a constitutive law for the ice.

![Fig. 2. Typical time history of power entering ice cover from atmosphere and transmitted into ocean.](image)
Example problem

The example represents an idealized, but realistic, description of winter conditions in the Beaufort Sea off the north slope of Alaska. The wind field is taken to be uniform and of magnitude 0.4 Pa (see Fig. 3), and an elastic-plastic constitutive law is assumed for the ice.

When a uniform wind field exists, the behavior of a long strip of ice \(0 < x < l\) may be assumed to be one-dimensional (see Fig. 3). That is, the velocity, stress, and thickness distribution vary with time and only one spatial direction, \(x\) (Pritchard and Schwaegler, 1976). The ice satisfies an elastic-plastic constitutive law similar to that by Coon and others (1974) and Pritchard (1975). The yield surface has been modified to be a diamond (Pritchard, 1978). The normal flow rule and elastic response are retained. Yield strength in isotropic compression depends on the ice-thickness distribution (Coon and others, 1974; Thorndike and others, 1975). However, material functions in the ice-redistribution function have been modified to provide yield strengths that are known to occur during winter in the Beaufort Sea. To accommodate these modifications, it was required that the energy dissipated by shear ridging that does not alter the thickness distribution be accounted for, in addition to the dissipation by gravitational potential energy and frictional sliding (Rothrock, 1975(a)). A complete description of the mathematical equations is too lengthy to include in the body of this paper. A similar version of the model has been presented by M. D. Coon, R. S. Pritchard, and E. Leavitt, and it will appear in the AIDJEX final report to the National Science Foundation. The material parameters are identified in the Appendix to this paper.

The oceanic boundary layer is modeled by a quadratic drag law:

\[
\tau_w = \rho_w c_w |\mathbf{v} - \mathbf{v}_g| \mathbf{B}(\mathbf{v} - \mathbf{v}_g),
\]

where \(\rho_w c_w = 8.0 \text{ kg/m}^3\) and \(\mathbf{B}\) represents a counterclockwise rotation through the angle \(\pi + \beta\) where \(\beta = 20^\circ\). The geostrophic ocean current \(\mathbf{v}_g\) and the ocean tilt \(mg \mathbf{V} \mathbf{H}\) have been neglected. These terms are felt to be unnecessary for understanding the basic concepts introduced in this paper.

The ice is assumed to be initially at rest and stress-free. The air stress is applied as a step at \(t = 0\). The initial thickness distribution is shown in Table I and thermodynamic growth rates are typical for 1 January (Coon and others, 1974).

The velocity field is presented in Figure 4. The onshore component is approximately twice as large as the alongshore component. A plastic region forms at the shore and builds outward with time and deformation. The region is 300 km wide after 1 d and 500 km wide after 4 d. The region beyond the plastic region, where velocity components are nearly uniform, remains elastic. At the right-hand boundary, the edge of the ice cover is seen to move leftward with time due to the general onshore motion. For this example, the motion outside the plastic boundary layer near shore is oriented approximately 30° to the right of onshore.
### Table I. Initial thickness distribution

<table>
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<tr>
<th>Thickness $h$ (m)</th>
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<td>0.805</td>
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<tr>
<td>10.00</td>
<td>1.000</td>
</tr>
</tbody>
</table>

#### Fig. 4. Velocity field at one-day intervals for example problem.
As the motion continues, deformation of the plastic region is accompanied by a reduction of the area covered by open water and thin ice. As a result, the ridging causes the ice strength to increase. This hardening process allows the stress divergence to increase and slows the ice motion. During the 4 d period, the far-field ice speed drops from approximately 0.18 m/s to 0.16 m/s.

The power density fields after 1 d and 4 d are shown in Figures 5 and 6. The two plots are similar qualitatively, with the changes in the values dictated by changes in velocity caused by hardening of the material. The free-drift atmospheric power-density input $P_{a\text{fd}}$ is nearly uniform and independent of time for the constant applied wind stress. The slight reduction near the shore is in response to an increase in the mass density which is a result of the deformation and ridging. The atmospheric power-density input $P_a$ is proportional to the onshore
component of the ice velocity since a constant onshore wind is applied. The power transmitted to the ocean may be rewritten for the case as

$$p_w = p_w c_w |v|^3 \cos \beta.$$  \hspace{1cm} (15)

Therefore, as the speed is halved, the power transmitted into the ocean drops by a factor of eight. This is reflected in the rapid decrease of $p_w$ in the plastic boundary layer.

The shaded areas in Figures 5 and 6 represent $p_1$ which is the difference between $p_u$ and $p_w$ (Equation (2)). It is seen that the effect of ice-stress divergence is felt throughout the region. Furthermore, at each point the effect increases with time as the material hardens. The effect of ice-stress divergence is presented directly in Figures 7 and 8 for days 1 and 4. Since these

![Fig. 7. Spatial power-density field from ice stress contribution and dissipation after one day.](image)

![Fig. 8. Spatial power-density field from ice stress contribution and dissipation after four days.](image)
fields go to zero at the left-hand boundary where the velocity is zero, it is evident that $p_l$ cannot be used alone to judge the effect of ice stress. As mentioned before, the entire effect of the ice cover is measured as $P_{afd} - P_w$. This term contains $p_l$ but also includes a contribution caused by differences between the free-drift and complete velocity fields.

Also shown on Figures 7 and 8 is the power dissipated by the ice through deformation $p_d$. It is seen that this dissipation is a maximum at the shore even though deformations are larger at a location in the plastic boundary layer away from shore. This fact occurs because the stress is largest at shore and so is the product of stress and stretching. All dissipation occurs in the plastic region. This region is seen to spread in area, and the magnitude of the power dissipation drops as the strength increases.

The power associated with divergence of the stress flux is the difference between $p_d$ and $p_l$ (Equation (6)). The sign of $p_l$ changes in the plastic region. We can interpret this change as a function of the direction of the transfer of energy by the stress flux. The horizontal transfer is from the elastic (rigid) region leftward toward the strong plastic region near the shore.

The average power terms over the entire region $\mathcal{R}$ are shown in Figure 9. The free-drift atmospheric input $P_{afd}$ is constant in time. The actual power input from the atmosphere $P_a$ is seen to decay with time. Since air stress is constant in time and space, this decay is caused by a decay in the spatial average of the onshore velocity component. The power transmitted into the ocean $P_w$ decays similarly. The total effect of the ice on energy transmitted into the ocean is $P_{afd} - P_w$. This difference increases with time as the material hardens, as did $P_{afd} - P_w$ at each point. It is seen that after 1 d approximately 75% of the available power of the atmosphere ($P_{afd}$) is input to the ice cover. Of this input power, 40% is dissipated by the ice while the remaining 60% is transmitted to the ocean. Stated another way, it is seen that 30% of the power available is dissipated by the ice cover while only 45% of the power available is transmitted into the ocean. After 4 d, 61% of the power available is input to the ice cover, while 34% and 27% of the power available are dissipated by the ice and transmitted to the ocean, respectively. For this example, the difference $P_a - P_w$ is the power dissipated by the ice $P_d$ because no work is done at the boundaries. Stated another way (see Figs 7 and 8) this means the area under $p_d$ equals the area under $p_l$ even though values are different at each point. In other words, all the energy in the ice is dissipated since it cannot be transmitted across the boundaries.

![Fig. 9. Average power density as a function of time.](image-url)
If this example is thought to be typical of conditions observed along the north slope of Alaska, then it is seen that the ice cover has a significant effect both on the amount of energy transmitted mechanically from the atmosphere into the ocean and on the location where the remaining energy is dissipated.

**CONCLUSION**

The mechanical energy (power) relationships developed and illustrated in this paper provide a powerful tool for understanding and demonstrating the dynamic behavior of sea ice. These relationships allow the ice behavior to be expressed in terms of scalar quantities rather than the more complex vector and tensor quantities of air stress, water stress, velocity, ice stress, and deformation. This has been accomplished by the suitable combination of the complex variables into meaningful energy variables such as the power of the atmosphere, ocean, and internal stress.

The local mechanical-energy balance equation is derived by forming the inner product of ice velocity and the momentum equation. A global balance is formulated by integrating the local balance equation over an arbitrary spatial region. Since inertia is negligible for problems that require resolution on scales of 1 d, the kinetic energy is also negligible in the mechanical energy balance. Therefore, the equation appears as a balance of power input at the top surface, power stored and dissipated by the ice, and power input to the ocean at the bottom surface.

The power input by the air stress acting on the moving ice cover is dissipated by the ice through deformation, transferred horizontally across the ice by the divergence of the stress flux and transmitted to the ocean both locally and at distant points. The local mechanical energy balance helps describe this important feature of horizontal energy transport by the stress flux divergence. The global form of the law, on the other hand, provides a simple measure of the effect of the ice cover over the entire region.

The power associated with free drift is introduced. It permits a clear understanding of how the strength of the ice cover can change the amount of energy which can be transmitted from the atmosphere to the ocean. For example, when the ice cover completely stops, there is no energy taken from the atmosphere; but, if winds are great, there is a significant amount of energy available. By comparing these quantities, the effect of the ice cover is seen.

The mechanism by which energy can be transferred from the atmosphere into the ice and subsequently transported long distances horizontally across the ice cover and then dissipated in a small localized region is shown in an example. This example is for an onshore wind and a plastic ice model. For this case no shear zone is developed. The development of a shear zone would be another example of how the work done over a large region of the ice cover could be dissipated in a small region. Although the example treats a plastic ice model, the mechanical energy balance is applicable for any material model.

**APPENDIX**

**Constitutive Law Used in Example Problem**

The elastic-plastic material response is of the general form introduced by Pritchard (1975). The entire model description is too lengthy to include in this paper. Therefore, a reader unfamiliar with it must use the references to understand the details. Since the notation is consistent in all the works referenced, only a minimal description of the variables is offered here. A diamond yield surface, as shown in Figure A-1, is assumed. The relationship between isotropic yield strength \( p^* \) and the thickness distribution are shown in a later paragraph. A normal flow rule is assumed:

\[
D_\phi = \frac{\phi}{\sigma^2} (\lambda > 0), \quad (A-1)
\]

where \( \phi \leq 0 \) is the yield constraint.
The ice redistribution function $\Psi$ is assumed in the general form of Coon and others (1974):

$$\Psi = \|D_p\|\left[(x_0(\theta) + x_e(\theta)) W_r\right].$$

(A-2)

The ridging coefficient $x_r$ is shown in Figure A-2. The opening coefficient is $x_0 = x_r + \cos \theta$. This use of the diamond yield surface and the ridging coefficient shown in Figure A-2 requires that a shear energy sink be assumed. The ridging function $W_r$ follows Coon and others (1974) with $k = 15$, but $B(G)$ is modified as shown in Figure A-3.

The energy dissipated during ridging by gravitational energy and frictional sliding follows Rothrock (1975a). The yield strength is

$$p^* = c^* \int_0^\infty h^2 \sigma(h) \, dh,$$

(A-3)

and the coefficient $c^*$ is

$$c^* = \frac{1}{2} \frac{\rho g k}{\tan \phi} \left[1 + \frac{\mu'}{\rho \tan \phi} \left(\frac{\rho}{\rho_w} (k-1)\right)\right],$$

(A-4)

where $\rho = 900 \text{ kg/m}^3$, $\rho_w = 1000 \text{ kg/m}^3$, $g = 9.8 \text{ m/s}^2$, $\mu' = 0.35$, and $\phi = 39^\circ$ have been assumed.
REFERENCES


DISCUSSION

D. A. ROTHROCK: Is it obvious that the energy dissipated mechanically is negligible compared to the terms in the heat balance, so that the mechanics does not affect the heat balance?

R. S. PRITCHARD: We have not previously made a comparison. However, according to Maykut (1976) the wintertime incident radiation is on the order of 200 W/m². This compares with the power dissipated by deformation which is \( c \times 20 \text{ mW/m²} \). Therefore, the thermal input is about four orders of magnitude larger than mechanical dissipation and there can be no significant coupling. Furthermore, the mechanisms allowing conversion to thermal energy might allow only a small fraction of the mechanical power dissipated to be converted into heat.

W. F. BUDD: When ice is deformed to about 1%, about 30% of the total strain may be recoverable but for large strains in excess of 10% the proportion of the strain that can be recovered becomes negligible. This seems to apply in the sea-ice deformation where the bulk of the deformation is concentrated in regions of high percentage strain.

PRITCHARD: We agree. It is the belief of scientists who developed the AIDJEX ice model that elastic strains are unimportant for simulating sea-ice dynamics on these length and time scales. The maximum rate of change of elastic energy estimated from \( p_e = p^e/\tau \) when \( p^e = 10^4 \text{ N m}^{-1} \) is a very large stress, and \( e = 0.002 \) is a large elastic strain. If the time resolution is \( \tau = 10^5 \text{ s} \), then \( p_e \approx 2 \text{ mW/m²} \). Since plastic (or total) stretching is on the order of \( 2 \times 10^{-7} \text{ s}^{-1} \) and \( p_d \approx 20 \text{ mW/m²} \), the change in elastic energy is approximately 10% of the power dissipated by deformation. Of course, when time scales are shorter the elastic energy can be significant.
W. D. Hibler III: It appeared in one of your figures that plastic deformation was taking place over a region of \( c. 300 \text{ km} \). I would expect that initially the deformation should take place in only one grid cell. Was this the case? Also what is your resolution?

Pritchard: The results were presented after deformations had occurred for several days and the cells closest to shore had hardened. Initially, when strength was uniform, deformations were concentrated in the cell nearest shore. Grid spacing was 50 km for this calculation.

L. W. Morland: You remark that the analysis does not depend on the constitutive law, but the strain-working term \((\sigma\mathbf{D})\) must vary for different laws. If not negligible, and you suggest it is 40\% of the other terms, the evaluation appears to require the constitutive law and momentum-balance solution.

Pritchard: You are right, of course! The value of \( \rho_d \) that would be obtained would differ for various constitutive laws. We meant that the concept of using the various energy terms to evaluate the ice conditions and model performance is applicable for any constitutive law.

REFERENCE