ABSTRACT. The sliding law is defined as a basal boundary condition for the large-scale bulk ice flow, relating the tangential traction $t_b$ overburden pressure $p_b$, and tangential velocity $u_b$ on a smoothed-out mean bed contour. This effective bed is a lower boundary viewed on the scale of the bulk ice flow and is not the physical ice-rock or sediment interface. The sliding relation reflects on the same scale the complex motion taking place in the neighbourhood of the physical interface. The isostructural steady-state ice-sheet analyses of Morland and Smith (1980, 1982) have been shown surface profiles from the Greenland ice sheet and Devon Island ice cap, with their corresponding mass-balance distributions, to determine $t_b, p_b$, and $u_b$ for each case. These basal estimates are used in turn to construct, using least-squares correlation, polynomial representations for the well-known dependence $f(p_b)$ in the adopted form of sliding law $t_b = f(p_b)u_b$ with $n > 1$.

The two different data sets determine functions $f(p)$ of very different magnitudes, reflecting very different basal conditions. A universal sliding law must therefore contain more general dependence on basal conditions, but the two relations determined appear to describe the two extremes. Hence use of both relations in turn to determine profiles compatible with given mass-balance distributions can be expected to yield extremes of the possible profiles, and further to show the sensitivity of profile form to variation of the sliding relation. The theory is designed as a basis for reconstruction of former ice sheets and their dynamics which are related to the two fundamental determinants of surface mass balance and basal boundary condition.

INTRODUCTION

Direct observations suggest that glaciers whose soles are at the melting point slide directly over their beds (e.g. Kamb and LaChapelle, 1964; Peterson, 1970; Boulton and others, 1979; Vivian, 1980), or that an equivalent subglacial motion takes place within an underlying layer of deforming sediment (Boulton, 1979). However, it is apparent that in most cases there is no such subglacial dislocation, but that the glacier sole adheres to its bed (Goldthwait, 1960; Holdsworth, 1974(b)). However, in this case an "apparent bed" may occur as a well-defined shear plane within the ice immediately above a basal boundary layer of ice (Holdsworth, 1974(b); Boulton, 1972).

It is supposed that sliding or dislocation beneath temperate ice occurs because of the presence of a film of water between it and a bedrock surface, or because of high pore-water pressures in un lithified sediments which reduce their frictional resistance and allow them to deform. As yet, theories of basal ice movement have largely been developed from Weertman's (1957, 1964) model of temperate ice sliding over a bedrock surface (recently reviewed by Weertman, 1979; see also Liboutry, 1979).

The sliding law is a basal boundary condition for the large-scale bulk ice flow, and relates the tangential traction $t_b$ on a (smoothed-out) bed contour to the tangential velocity $u_b$ of the ice at this boundary in the same direction. Strictly $t_b$ and $u_b$ should be vectors in a tangent plane to allow a component of traction normal to the local sliding direction, but three-dimensional sliding theories tacitly assume that $t_b$ and $u_b$ are parallel. Negligible sliding velocity at this apparent bed, reflecting non-slip at the neighbouring real bed, must be a possible result.

The classical sliding theory introduced and extended by Weertman (1957, 1964) considers a bed consistent...
Weertman derives a sliding velocity from the two components of regelation slip and plastic flow past the obstacles for a given basal shear stress. The resulting law has the form

\[ u_b = B_{rb}(n+1)/2 \tag{1} \]

where \( n \) is the exponent in the creep law deduced by Glen (1955) from uniaxial compression tests. Lilbourne (1968) deduced a similar law but with disagreement over the appropriate measure of bed roughness and the magnitude of \( B \). The form of the sliding law in Equation (1), with \( n = 1 \), has been supported by Nye (1969) and Morland (1976) for the Newtonian-fluid approximation, but the significant non-linear viscous response of ice defies similar treatment. The simplifications inherent in these formulations have, however, made it difficult to test the law in the field.

Fowler (1979, 1981), in his analysis of sliding, defines two roughness parameters: the bed asperity \( \alpha \), defined as the bed roughness wavelength \( \lambda \) relative to ice depth \( d \), and the bedrock roughness slope \( v \), defined as roughness amplitude \( \eta \) relative to wavelength \( \lambda \) (Equations (2) and (3)) for the Newtonian assumption (Equation (3)) and further extends the Newtonian assumption (Equation (1)), and further

\[ u_b = n \alpha v^{-m/(1+n)} (\tau_b/R)^1/n \tag{2} \]

Equation (2) can also be written

\[ \tau_b = K_d u_b^{1/m} \tag{3} \]

where \( d \) is the ice thickness, although Fowler’s analysis applies only to \( d \gg \lambda \), the bed roughness wavelength, so does not allow the limit \( d \to 0 \). A simple extension of Equation (3) allows variation of the thickness \( d \) as \( d \to 0 \) in a limit smaller than the Greenland ice sheet. As expected, the magnitudes of the compressible coefficients \( \lambda(p_b) \) are very different, reflecting a different set of basal conditions. We suggest, though, that these two examples represent fairly extreme conditions, so that profile reconstructions using both relations would determine appropriate outer bounds. Naturally, the choice of \( m \) influences the pressure dependence of \( \lambda \). We also find that there is no sensible \( \delta(u_b) \) in the alternative single-function relation

\[ \tau_b = \lambda(p_b) u_b^{1/m} \tag{6} \]

with one arbitrary function \( \lambda(p_b) \) to determine from individual ice-sheet data for different values of an integer exponent \( m \). The correlation is carried out for two very different ice masses. First, the Expedition Glaciologique Internationale au Groenland (EGIG) profile of the present Greenland ice sheet (Holtzschuer and Bauer, 1956; Hofmann, 1974). Secondly, the west profile of the Devon Island ice cap (Hyndman, 1969-70; Moller, 1977, p. 147-54), which is an order of magnitude smaller than the Greenland ice sheet. As expected, the magnitudes of the respective coefficients \( \lambda(p_b) \) are very different, reflecting a different set of basal conditions. We suggest, though, that these two examples represent fairly extreme conditions, so that profile reconstructions using both relations would determine appropriate outer bounds. Naturally, the choice of \( m \) influences the pressure dependence of \( \lambda \). We also find that there is no sensible \( \delta(u_b) \) in the alternative single-function relation

\[ \tau_b = \lambda(p_b) u_b^{1/m} \tag{6} \]

which is consistent with the Greenland data. It is understood that our data correlation does not confirm a sliding relation of the form of Equation (6), but simply determines a basal boundary condition for the unmodified boundary of the global flow, which reconstructs the known profile from the given accumulation data.

It is known that significant temperature variation occurs in cold ice shution areas. Data from an isothermal approximation allows only the specification of a constant temperature in the rate factor, and we recognize that a proper thermomechanical analysis could yield a different distribution of basal velocity, and hence correlate with different coefficients \( \lambda(p_b) \). In particular, the calculated basal sliding velocity up on the smoothed bed may well be reduced if enhanced velocity gradients in
warmer zones increase the differential velocity between the surface and bed. Alternatively, if the enhancement occurs mainly in a thin thermal boundary layer below the adopted smooth bed, as proposed in Nye's (1959) pioneering ice-sheet analysis, then the basal sliding velocity so defined would be increased.

The polynomial relation exhibits bounded viscosity at zero stress, unlike a power law with exponent $n > 1$, and correlates much closer to the same data (Smith and Morland, 1981) from Glen's (1955) uniaxial compression data at $T = 273.13$ K, noting that $a(273.13)$ is approximately unity for Equations (11) and (12). Following the MJ notation

$$w(J_2) = \frac{3}{2} \left[ (c_0 + 3c_1 J_2^2 + 9c_2 J_2^4) \right]$$

with

$$c_0 = 0.2224, \ c_1 = 0.07111, \ c_2 = 0.002195$$

when $\sigma_0 = 10^6$ N m$^{-2}$, $D_0 = 1 \text{ a}^{-1}$.

The dimensional results are considerably lower than the above laboratory and ice-shelf data (Thomas, 1971, 1973; Holdsworth, 1974) and correlates much closer to the same data (Smith and Shoemaker, 1982) than the polynomial to the lead-order ordinary differential equation subject to initial (margin) value and slope. The dimensionless normalized variables and analysis depend on $\epsilon$ which is determined by individual sheet conditions, but we wish to express the relations and solution in common normalized stress and velocity variables for application to any ice sheet. Accordingly, we start from the lead-order solution expressed in physical variables (Morland and Johnson, 1982), but without eliminating the basal velocity $u_b$ by the sliding law. That is

$$\rho = -\alpha_{xx} = -\alpha_{yy} = \rho g(h - y), \ \rho b = \rho g(h - f),$$

$$\alpha_{xy} = -}\rho g h'(h - y), \ \alpha_b = -\rho g h'(h - f),$$

$$u = \frac{\alpha_{bb}}{\rho g h'} \left[ \frac{\alpha_{b}}{\alpha_0} \right] - \frac{\alpha_{b}}{\alpha_0} \left( \frac{h - f}{\alpha_0} \right)$$

where

$$\epsilon = \text{sgn}(h'), \ \frac{\alpha_{b}}{\alpha_0} = \frac{3}{2} \epsilon \text{ct}^2 + \frac{3}{4} \epsilon \text{ct}^4 + \frac{3}{2} \epsilon \text{ct}^6.$$  

Note that $\epsilon$ denotes $\alpha_{xy}$ and $\epsilon_0$. The profile equation is

$$\frac{\partial}{\partial t} \left( \frac{h - f}{\alpha_0} \right) + \frac{\alpha_{bb}}{\rho g h'} \left[ \frac{\alpha_{b}}{\alpha_0} \right] - \frac{\alpha_{b}}{\alpha_0} \left( \frac{h - f}{\alpha_0} \right) = q - b \ (18)$$


\[\text{Fig. 1. Ice-sheet cross-section.}\]

$y = f(x)$ is restricted to small slope $f'(x)$, and small mean bed inclination to the horizontal can be incorporated in the function $f(x)$. The ice sheet has a free surface $y = h(x)$, on which the normal and tangential tractions $t_y$ and $t_x$ are zero, and on which there is an accumulation distribution $q$ defined as the volume flux of ice per unit horizontal cross-section entering the sheet. Negative $q$ denotes ablation.

Basal drainage $b$ on $y = f(x)$ is the volume flux of ice leaving the sheet per unit horizontal cross-section. Let $q_m$ denote a maximum accumulation (ablation) magnitude, and let $h_0$ be a sheet thickness magnitude and $\epsilon_0$ a semi-span magnitude.

We assume that the ice behaves like an incompressible non-linear viscous fluid on gravity-driven-flow time scales, and satisfies a constitutive relation

$$D = D_0 a(T) \frac{\partial J_2}{\partial s} S$$

where $D$ is the strain-rate tensor, $S$ is a dimensionless deviatoric stress tensor defined by

$$S = \sigma_0^{-1} (u + p) + \frac{1}{3} \text{tr } u,$$

and

$$J_2 = \frac{1}{3} \text{tr } S^2,$$

and where $D_0$ and $\sigma_0$ are strain-rate and stress units respectively and $I$ is the unit tensor. The temperature-dependent rate factor $a(T)$ becomes a constant defined by its value at the chosen temperature in the isothermal approximation. We adopt the factor

$$a(T) = a_1 \exp \left( \frac{a_2}{T} \right) + a_2 \exp \left( \frac{a_3}{T} \right)$$

where

$$T = 273.15 \ K + 20 \ K \ T,$$

and

$$a_1 = 0.7242, \ a_2 = 11.9567, \ a_3 = 0.3438, \ a_2 = 2.9494,$$

which is a close correlation constructed by Smith and Morland (1981) to the Mellor and Testa (1969) uni-

\[\text{Fig. 1. Ice-sheet cross-section.}\]
where
\[ \Pi(t) = c_0^2 + \frac{9}{5} c_1 t^2 + \frac{27}{7} c_2 t^5, \] (19)
subject to the margin conditions
\[
\text{margin: } h-f = 0, c(h'-f')(-cH')^m = \left( \frac{\lambda_0}{\alpha g} \right)^m (q_m - b_0), \] (20)
where
\[ \lambda = \lambda_0 (h-f), \quad q = q_0, \quad b = b_0 \text{ as } h-f = 0. \] (21)
The \( g_1(t) \) and \( a(t) \) definitions are more convenient than those of \( q, \dot{u} \) in MJ which contain the rate factor \( a(T) \), both \( g_1(t) \) and \( a(t) \) are order unity when \( t \) is order unity, corresponding to deviatoric stress of order \( 10^5 \text{ N m}^{-2} \).

Now an order of magnitude for the basal shear stress \( \tau_b \) is the stress unit \( \epsilon_0 \cdot 10^5 \text{ N m}^{-2} \), while the longitudinal velocity is greater by a factor \( \epsilon^{-1} \), where \( \epsilon \) is a surface-slope magnitude.

Similarly, the normal velocity \( v \) has magnitude \( q_n \), while the longitudinal velocity is greater by a factor \( \epsilon^{-1} \), which follows directly from the mass balance (MJ). We introduce common normalized variables in terms of a fixed slope magnitude
\[ \epsilon_0 = 0.005, \] (22)
and depth and semi-span related by
\[ h_0 = \epsilon_0 c_0, \] (23)
so that \( h_0 \) is determined directly \( \epsilon_0 \) is specified. Thus
\[
(q, q_0, b) = (q_0, h_0, b_0), \quad (u, u_0) = (q_0, h_0, u_0), \quad \epsilon = h_0 Y, \quad h(x) = h_0 H(x), \quad f(x) = h_0 F(x), \] (24)
and the dimensionless pressure \( \bar{p} \) has a unit \( p = \epsilon_0 c_0^2 = 10^5 \text{ N m}^{-2} \) with the values given in Equations (15) and (22). Choosing an accumulation and normal velocity magnitude
\[ q_m = 1 \text{ m a}^{-1}, \] (25)
the dimensionless longitudinal velocity \( \dot{u} \) has a unit 200 m a\(^{-1}\).

In these dimensionless variables
\[ p_b = k(H-F), \quad \tau_b = -\epsilon_0 \epsilon k H^2(H-F) \] (26)
where
\[ k = \frac{\rho g h_0 \epsilon_0}{a_0} \] (27)
depends on the value of \( h_0 \) given by Equation (23), that is, on the prescribed semi-span \( \epsilon_0 \). The profile equation becomes
\[
\frac{d}{dx} \left( \frac{(H-F) \dot{u}}{a_0^2} + a_0 \rho \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \frac{d}{dx} \right) \] (28)
with margin conditions
\[
\text{margin: } H-F = 0, c(h'-f')(-cH')^m = \left( \frac{\lambda_0}{\alpha g} \right)^m (q_m - b_0), \] (29)
again depending on \( h_0 \). We express the basal sliding Equation (6) in the form
\[
\tau_b = \lambda(p_b) \bar{u} \dot{u} \] (30)
where the required linear dependence of \( \lambda(p_b) \) as \( p_b \to 0 \) is imposed by demanding that \( u(p_b) \) is analytic and bounded near \( p_b = 0 \), with
\[
\lambda_0 = \frac{\lambda_0}{\epsilon_0} \left( \frac{q_m}{\epsilon_0} \right)^{1/m}, \] (31)
so the coefficient of \( (q_m - b_0) \) in the second margin condition (29) is simply \( \epsilon_0 q_m \).

The MJ solution is obtained by applying a given sliding law (30) to eliminate \( \dot{u} \), and solving the consequent second-order non-linear differential equation (28), subject to initial conditions (29), for the profile \( H(X) \). Here we apply Equation (28) to a given profile \( H(X) \) with associated surface accumulation/ablation and basal drainage \( q_b \) to evaluate \( \dot{u}(X) \). With \( p_b(X), \tau_b(X) \) given by Equations (26) we can, in principle, determine one function \( \lambda(p_b) \) in a sliding relation (30).

GREENLAND ICE SHEET PROFILE (EGIG)

In order to determine the basal (slip) velocity \( \dot{u} \) from Equation (28) we need to know the surface profile \( H(X) \), the bed profile \( F(X) \) and the net accumulation/ablation distribution \( q(X) \). We assume henceforth that there is zero drainage at the bed, \( B = 0 \) in Equation (28), since actual values are expected to be less than 0.01 m a\(^{-1}\) (Boulton, 1983). The physical data for the surface and bed profile is taken from Hofmann (1974), with a corrected "near margin" profile from Holtzscherer and Bauer ([1956]). These data, along with a 7-degree Chebyshev polynomial approximation, appears in Figure 2a. The bed form for the EGIG profile is a series of undulations about sea-level, \( h = 0 \), see e.g. Boulton, 1983. We have investigated both this form and a flat bed at sea-level, obtaining similar global results. Detailed results based on the flat bed assumption are now presented. The accumulation/ablation data (Hofmann, 1974; personal communication from L.D. Williams in 1982) is shown in Figure 2b along with the corresponding 7-degree Chebyshev polynomial approximation. We choose \( \epsilon_0 \) to be the actual distance between the divide and the margin which are then represented by end points \((0, 1)\) in the dimensionless co-ordinate \( \chi \). Here
\[ \epsilon_0 = 420 \text{ km}, \quad h_0 = 2.1 \text{ km}. \] (32)
The accumulation data give
\[ \int_0^1 q d\chi = 0.009, \] (33)
instead of zero required by the steady-state comparison, but this value is extremely small in comparison with both the maximum normalized ablation value 3.75 and the maximum normalized accumulation value 0.52.

A small adjustment to the data is therefore required to obtain a steady-state pattern; a convenient form is to replace \( \lambda(H) \) by \( \lambda_0(H) \) where
\[
\int_0^1 q d\chi = 0.009, \quad \epsilon_0 = 420 \text{ km}, \quad h_0 = 2.1 \text{ km}. \] (34)
which can be interpreted as a small basal drainage component linearly dependent on altitude in Equation (28). If a function \( \lambda(H) \) is prescribed, then Equation (28) is directly integrable and an alternative adjustment...
may be used. Both forms (34) and (35) were used in the numerical calculations and the results were indistinguishable. The results given in this section are based on the adjustment (34).

Using the polynomial representations of $H(x)$ and $Q(x)$, or $Q(x)$, the differential equation (28) gives the dimensionless basal velocity $u_b(x)$ displayed in Figure 3, along with the dimensionless pressure $P_b$ and dimensionless shear traction $T_b$ given by Equation (26). Also shown is the normalized surface slope $\theta_b/P_b$. The three velocity curves in Figure 3 correspond to three different uniform temperatures $T = -30°C$, $-26°C$, $-23°C$, using the temperature-dependent rate factor $a(T)$ given by Equations (11) and (12); $a(-23°C) = 10^{-2}$, $a(-30°C) = 4 \times 10^{-3}$. These are estimates of mean temperature in the bulk of the inner ice mass (Boulton, 1983). The basal velocities in this example increase monotonically with distance from the divide, reaching a value equivalent to approximately 130 m a$^{-1}$ at the margin. Discrepancies from actual values are expected on account of the assumption of a flat bed, and such discrepancies are more noticeable near the margin. The lower the ice temperature the less is the internal deformation, which, for a fixed glacier surface profile and accumulation/ablation distribution, must produce a compensating increase in the predicted basal velocity $u_b$. However, the influence of temperature in the isothermal approximation on the predicted basal velocity is seen to be negligible in this example. Temperature variation with depth and longitudinal distance may have a more significant effect. Note, though, that in this isothermal approximation, the basal velocity required as a boundary condition for the global flow is not negligible over a major part of the bed, and could not be described by a non-slip condition on the smooth apparent bed.

We now adopt the minimum temperature $T = -30°C$ of the above set, which represents a natural bound of

\[ \frac{\theta_b}{P_b} \]

\[ \theta_b = \frac{1}{0} dX \]

\[ \theta_b = \frac{1}{0} dX \]

Fig. 3. Distributions of normalized basal pressure $P_b$, shear traction $T_b$, tangential velocity $u_b$ and surface slope determined by $u_b/P_b$ (Greenland profile). The three $u_b$ curves correspond to $T = -30°C$ (---), $-26°C$ (---), $-23°C$ (---). The units of $P_b$, $T_b$, $u_b$ are respectively $200 \times 10^5$ N m$^{-2}$, $10^5$ N m$^{-2}$, 200 m a$^{-1}$.

Fig. 4. Variation of $u_b^{1/m}$ with surface slope $\theta_b/P_b$ for different values of $m$ (Greenland profile).
mean temperature, to produce the scaled-down strain-rates expected in natural ice flow as compared to laboratory creep tests (see earlier discussion on page 133). First we investigate the changes in the function \( A(P_b) \) when different values of \( m \) are chosen in Equation (30). Figure 4 shows \( u_b/\beta \) versus dimensionless slope \( T_b/P_b \) while Figure 5 shows the function \( \lambda(P_b) = T_b/\beta P_b^{1/m} \) versus \( P_b \) for \( m = 1, 2, 3, 4 \) (Greenland profile). In both figures the results for \( m = 2, 3, 4 \) are not strongly dissimilar whereas the results for \( m = 1 \) have distinct features. In Figure 4 it is the behaviour of \( u_b/\beta \) as \( T_b/P_b \to 0 \), and in Figure 5 it is the behaviour of \( \lambda(P_b) \) as \( P_b \to 0 \), both limits occurring at the ice divide. Both figures reflect the limit behaviour at the divide of \( T_b/\beta \) approaching a non-zero finite value and \( T_b/\beta P_b^{1/m} (m > 1) \) approaching zero as \( T_b \) and \( \beta P_b \) approach zero. The function \( \lambda(P_b) \) versus \( P_b \) is shown for different values of \( m \) in Figure 5. It is evident that \( \lambda(P_b) \) is very closely linear, and with the same gradient, for each value of \( m \) including \( m = 1 \), until the basal pressure \( P_b \) reaches half its maximum value 1.3 which is attained at the divide. Thus \( \lambda(P_b) \) is accurately represented by a quadratic over this range.

We now focus attention on the case \( m = 1 \), which guarantees a unique surface slope at the margin when ablation occurs there (Morland and Johnson, 1982). For convenient application of a sliding relation such as the second of Equations (30), we require an explicit representation of the function \( \mu(P_b) \) over the entire range of \( P_b \). A linear form of \( \mu(P_b) \) is assumed for \( 0 < P_b < 0.7 \), while for \( 0.7 < P_b < 1.3 \) a polynomial representation is used which satisfies continuity of \( \mu \), its first and second derivatives at \( P_b = 0.7 \). Correlation by least squares is applied for increasing degree of polynomial until the resulting function is in good agreement with data, and not distinguishable in the graphical form on the scale of Figure 6. The result is

\[
\begin{align*}
\mu(P_b) = 9.000 & - 6.657 P_b, \quad 0 < P_b < 0.7, \\
5 & \\
\mu(P_b) = \sum_{r=0}^{5} \mu_r P_b^r, \quad 0.7 < P_b < 1.3, \\
\mu_0 = -53.596, & \mu_1 = 253.643, \\
\mu_2 = -324.134, & \mu_3 = 26.753, \\
\mu_4 = 176.028, & \mu_5 = -72.761.
\end{align*}
\]

However, the continuation of this polynomial representation starts to decrease for \( P_b > 1.49 \). Ice-sheet profiles generated with this sliding relation for which the pressure significantly exceeds this value so that \( \mu \) returns to its low-pressure values would be physically unsatisfactory. Hence, beyond a pressure \( P_m \) the polynomial form is replaced by a linear extension

\[
\mu(P_b) = 19.73 + 54.43(P_b - P_m), \quad P_b > P_m = 1.3,
\]

which satisfies continuity of \( \mu \) and \( \mu' \) at \( P_b = P_m \). This latter transition value \( P_m \) is arbitrary but it is chosen near the point of inflexion of the polynomial given by the second of Equations (36), i.e. the value of \( P \) at which the slope \( \mu'(P_b) \) starts to decrease.

The accuracy of the sliding-relation construction (36) can be verified by solving the ordinary differential equation (28) for the EGIG profile \( H(X) \) with the given accumulation/ablation distribution shown in Figure 2b, with the adjustment (34). A graphical comparison with the original profile (Fig. 2a) shows no distinction. This applies to both Equations (36) and (36) amended by (37) since \( P_b \) does not significantly exceed the transition pressure \( P_m \).

**NORTH-WEST DEVON ISLAND ICE CAP PROFILE**

The procedure outlined in the previous section is now repeated for the much smaller north-west profile of the Devon Island ice cap. The surface profile data
are extracted from Hyndman (1965), which also contains estimates of the bed profile determined by gravity measurements for various cross-sections of the ice cap. Once again calculations are made for both a polynomial representation of the bed based on the available data, and on a simplified version (F varying linearly with H) which reflects similar global results. The results of the simplified version are presented.

The accumulation/ablation data for the north-west profile are taken from Müller (1977, p. 147-54). A weighted mean of these data, which represent measurements for the years 1960 to 1975 inclusive, is determined in order to achieve an accumulation/ablation pattern which is as nearly as possible in steady state with the assumed profile. Although the margin of the ice cap is approximately 600 m above sea-level over the island, the accumulation/ablation data shown in Figure 7b is for altitudes ranging from the maximum value of 1800 m at the divide down to zero altitude. The lower-altitude data (h < 600 m) represent measurements on one of the outlet glaciers at the extreme north-west of the ice cap, an estimate of which must be included in the analysis in order to approximate a steady-state system. In Figure 7a the surface profile data for the ice cap and glacier are shown, along with the corresponding 11-degree Chebyshev polynomial representation and the simplified version of the bed profile (\( F = 7H/9 \)) which reproduces measured depths near the summit (Paterson and Clarke, 1978).

Figure 7b shows the distribution of normalized basal pressure \( \bar{P}_b \), shear traction \( \tau_b \), tangential velocity \( u_b \), and surface slope determined by \( \tau_b/\bar{P}_b \) (Devon Island profile). The \( u_b \) curve corresponds to \( T = -30^\circ C \) (---). The units of \( P_b \), \( \tau_b \), \( u_b \) are respectively \( 100 \times 10^5 \text{ N m}^{-2} \), \( 200 \text{ m a}^{-1} \), \( 200 \text{ m a}^{-1} \).

are again performed for the assumed uniform temperature \( T = -30^\circ C \).

Figure 9 shows the behaviour of \( \Sigma (P_b) \) versus \( P_b \) for \( m = 1,2,3,4 \). Once again it is apparent that the case \( m = 1 \) is distinct from the cases \( m = 2,3,4 \) in the limiting behaviour of \( u_b/\bar{P}_b \) as both \( \bar{P}_b \) and \( u_b \) approach zero at the divide. Figure 10 shows \( \bar{\mu}(P_b) \) versus \( P_b \) for \( m = 1,2,3,4 \). We note the similarity in shape with the corresponding curves in Figure 6, with each again having closely linear sections, but with a different gradient. The vast difference in the range of \( \bar{\mu}(P_b) \) for the Greenland (Fig. 6) and Devon Island (Fig. 10) profiles is due mainly to the different scales of the basal pressure, \( P_b \leq 1.3 \) and \( P_b \leq 0.17 \) respectively, with the Greenland ice roughly eight times as thick as the Devon ice at corresponding values of distance \( x \). An approximate smooth representation of \( \bar{\mu}(P_b) \) for the \( m = 1 \) curve in Figure 10 is given by

\[
\bar{\mu}(P_b) = 1000 - 10000 P_b, \quad 0 < P_b < 0.08,
\]

\[
\bar{\mu}(P_b) = \sum_{r=0}^{3} \mu_r P_b^r, \quad 0.08 < P_b < 0.15,
\]

\[
\mu_0 = 1424, \quad \mu_1 = -17346, \quad \mu_2 = -15306, \quad \mu_3 = 510204
\]

\[
\bar{\mu}(P_b) = 200 + 12500(P_b-0.15), \quad P_b > 0.15
\]
DISCUSSION OF RESULTS

The steady-state isothermal solution of Morland and Johnson (1980, 1982), with different normalization, has been used to determine the basal tangential velocity $U_b$ on a smooth bed contour defining the lower boundary for the global flow. Calculations were made using the surface profile data and accumulation/ablation data from two very different examples of present-day ice masses, namely the Greenland ice sheet and the Devon Island ice cap. Normalized distributions of the basal pressure $P_b$, the basal tangential traction $T_b$ and the basal tangential velocity $U_b$ were determined, and used to find a "friction" parameter $\lambda(P_b)$ in an assumed sliding relation of the form of Equation (30) for $m = 1, 2, 3$, and 4.

It is found for each case that the behaviour of $\lambda(P_b)$ for $m = 2, 3$, and 4 is similar but that the $m = 1$ results are distinct. This phenomenon stems from the limiting value of $T_b/U_b$ as both $T_b$ and $U_b$ approach zero at the divide. For $m = 1$, the limit is finite and non-zero, while for $m > 1$, the limit is zero. These differences are evident in Figures 4, 5, and 9.

In general, $\lambda(P_b) = \lambda(P_b)/P_b$, initially decreases from a finite positive value as $P_b$ increases from zero then starts to increase from a mid-range value $P_b$ of $P_b$ (see Figs 6 and 10). The main differences between the two examples are

(i) for the Greenland profile $\lambda(P_b)$ is a monotonic function of $P_b$, while for the Devon Island ice cap it is not;

(ii) the much smaller range of the Devon Island ice cap basal pressure leads to dramatically larger values of $\lambda(P_b)$.

Such wide differences in magnitude of the function $\lambda$, and hence $\lambda$ in Equation (6), correspond to wide differences in the calculated basal sliding velocity at given basal shear stress. Bed structure and thermal conditions will influence the gross friction represented by the sliding law, but more significantly, bed mobility could significantly reduce the actual sliding velocity. Whereas it is the absolute basal ice velocity which appears in the differential equation (28), it is strictly the relative slip velocity which should enter the sliding relations (4), (6), and (7). Compatibility of the predicted Greenland and Devon basal velocity magnitudes, which approach 130 m a$^{-1}$ and 13 m a$^{-1}$ at the respective margins, in the common range of overburden pressure could require, though, a significant Greenland bed movement to account for such a large fraction of the calculated basal velocity.

Significant temperature profiles and strong creep dependence on temperature through $a(T)$ will influence the basal velocity calculation, and basal temperature could affect the sliding "friction" or influence bed mobility. Given an empirically deduced temperature field, it is possible to generalize the analysis to incorporate the rate factor $a(T(x,y))$, and we are now investigating such temperature effects on the basal velocity.

In principle, given ice-sheet data could be correlated to other forms of sliding relations: for example, the form of the first of Equations (4) with $\lambda(P_b)$ prescribed and $\mu(U_b)$ determined. More specifically, if we choose Equation (7) which satisfies the required asymptotic behaviour (5) as $P_b = 0$, then $\mu(U_b) = \mu(P_b)$ determined by the Greenland data is represented in the normalized variables $U_b$, $P_b$, $T_b$ by the curve for $m = 1$ in Figure 4. Note, however, the large gradient of the corresponding $P_b = \mu(P_b)$ when $T_b/P_b$ is near unity, which is associated with the rapid change of the sliding velocity $U_b$ while the surface slope changes little. That is, the sliding velocity is very sensitive to small changes of surface slope, and it would be difficult to represent this function $\mu$ accurately.

Unfortunately, data from a small set of ice sheets cannot determine a function of two variables such as $\tau_s(P_b, U_b)$, so we must start with restricted forms such as Equation (6). Equation (6) extended to include a temperature-dependent factor. It is clear that there is no universal sliding law, and choice of a basal sliding condition will depend on the particular application. Until thermal and bed-structure effects are established, a range of sliding conditions should be considered.
be considered to estimate their influence on solutions. Such a parameter study has been included in an investigation of equilibrium profiles in various environments by Boulton and others (1984).

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